Phenomenological template bank for Black hole coalescence waveforms

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Motivation

- Coalescing BH binaries are one of the most promising sources for ground/space-based IFOs.
- The inspiral and ring-down stages can be accurately modelled by approximation and perturbation techniques in GR.
- Signals from inspiral and ring-down stages can be probed via optimal filtering. So far, signals from merger part are probed only via non-optimal filters.
- For $M >\sim 50\ M_{\odot}$, only the merger+ring-down parts of the coalescence signal falls into the detection band of ground based detectors. This sets a limit on the mass of the binaries which can be probed through matched filtering techniques using *inspiral* templates.
- Similarly, there will be a lower limit on the mass for which pure ringdown templates can be used.
- Recent progress in Numerical Relativity in calculating the waveforms from BH mergers.

Plan

- Coherently search for all three stages of the BH coalescence signals through a single (phenomenological) template bank. The template waveforms should contain inspiral, merger and ring-down stages.

A single template bank for BH coalescence

- **Issue** How to construct a bank of templates?
  - May be too expensive to compute a bank of NR waveforms dense enough in the (m_1, m_2) parameter space.
  - An interpolated template bank - with parametrized templates.
- **Issue** How to construct the 'target' waveforms?
  - Need waveforms containing all three stages of binary coalescence - too expensive to (numerically) evolve the binary from very large separations.
  - Match PN inspiral waveforms with NR (merger+ring-down) waveforms.
- **Issue** How accurate/unique are the 'target' waveforms?
  - Different methods/initial data may predict slightly different waveforms - might not be unique for the same physical system.
  - Parametrisation allows flexibility for incorporating such differences.
- **Issue** How to lay down the templates?
  - Laying down the templates allowing a given mismatch.
  - The metric of the parameter space can be evaluated from the parametrized waveforms (or, directly from the numerical waveforms).

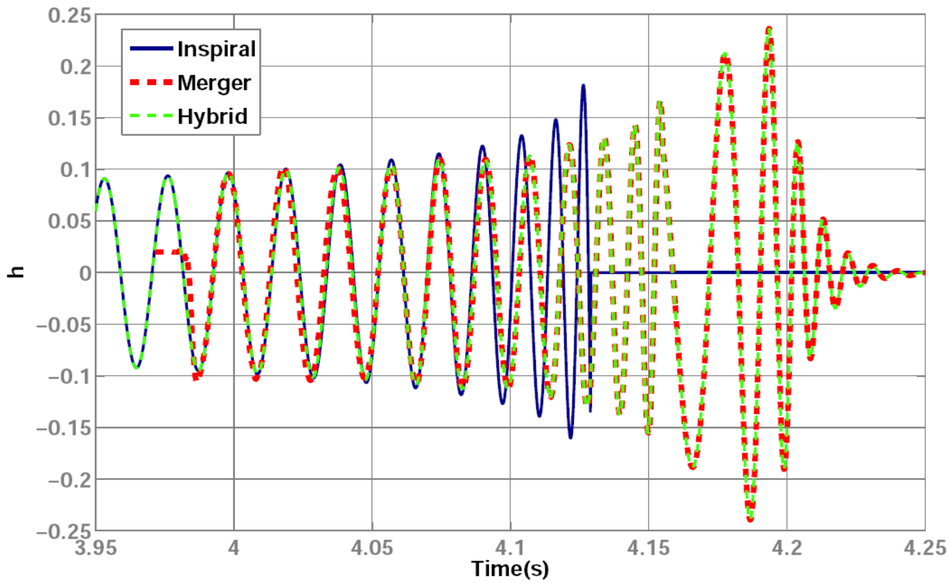
Matching PN and NR waveforms

- Minimize least square difference between PN (inspiral) and NR (merger+ring-down) waveforms over a matching region (a few cycles long), thus construct hybrid waveforms.

- Free parameters
  - Extrinsic: initial phase ϕ_0 of the inspiral wave, amplitude a of the merger wave, time-slide τ between the inspiral and merger waveforms.
  - Intrinsic: total mass M and mass-ratio $R = m_1/m_2$.

- Check internal consistency of the matching procedure by calculating overlaps between hybrid waveforms generated with different matching regions.

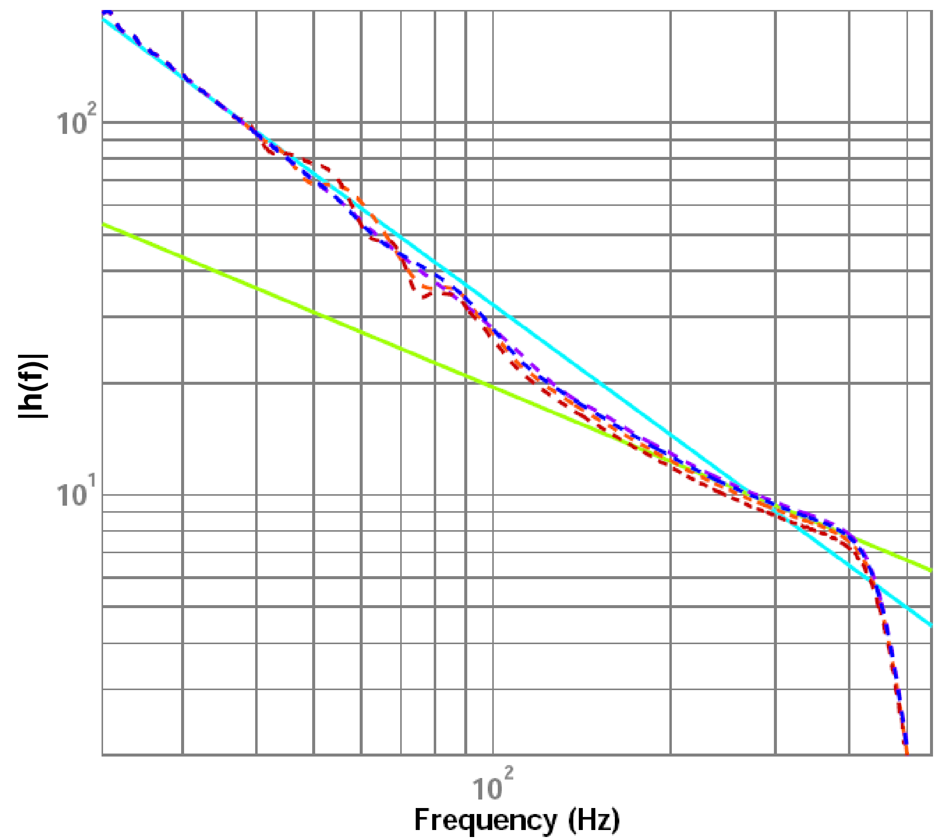
Matching PN and NR waveforms



Inspiral merger and 'hybrid' waveforms from $M = 40M_{\odot}$, $R = 1$ binary.

Inspiral waveforms: 3.5PN in phase, 'restricted' PN, TaylorT1.

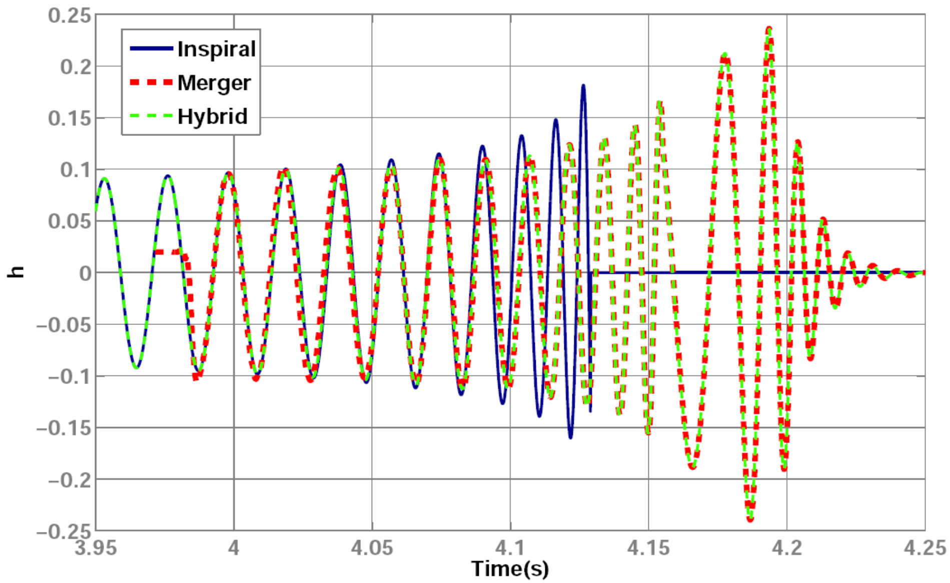
Merger waveforms: Equal mass (f500) simulation of the AEI group.



Fourier domain magnitude of the 'hybrid' waveforms constructed using different matching regions.

Merger waveforms used: Equal mass (d5) simulation of the AEI group.

Matching PN and NR waveforms



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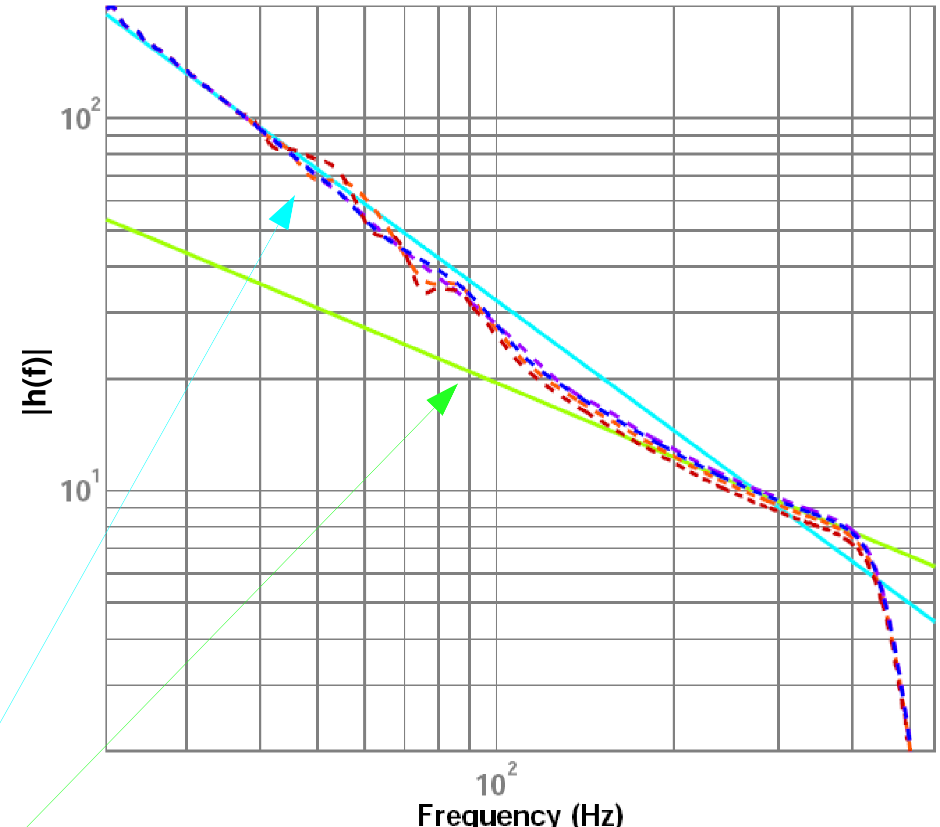


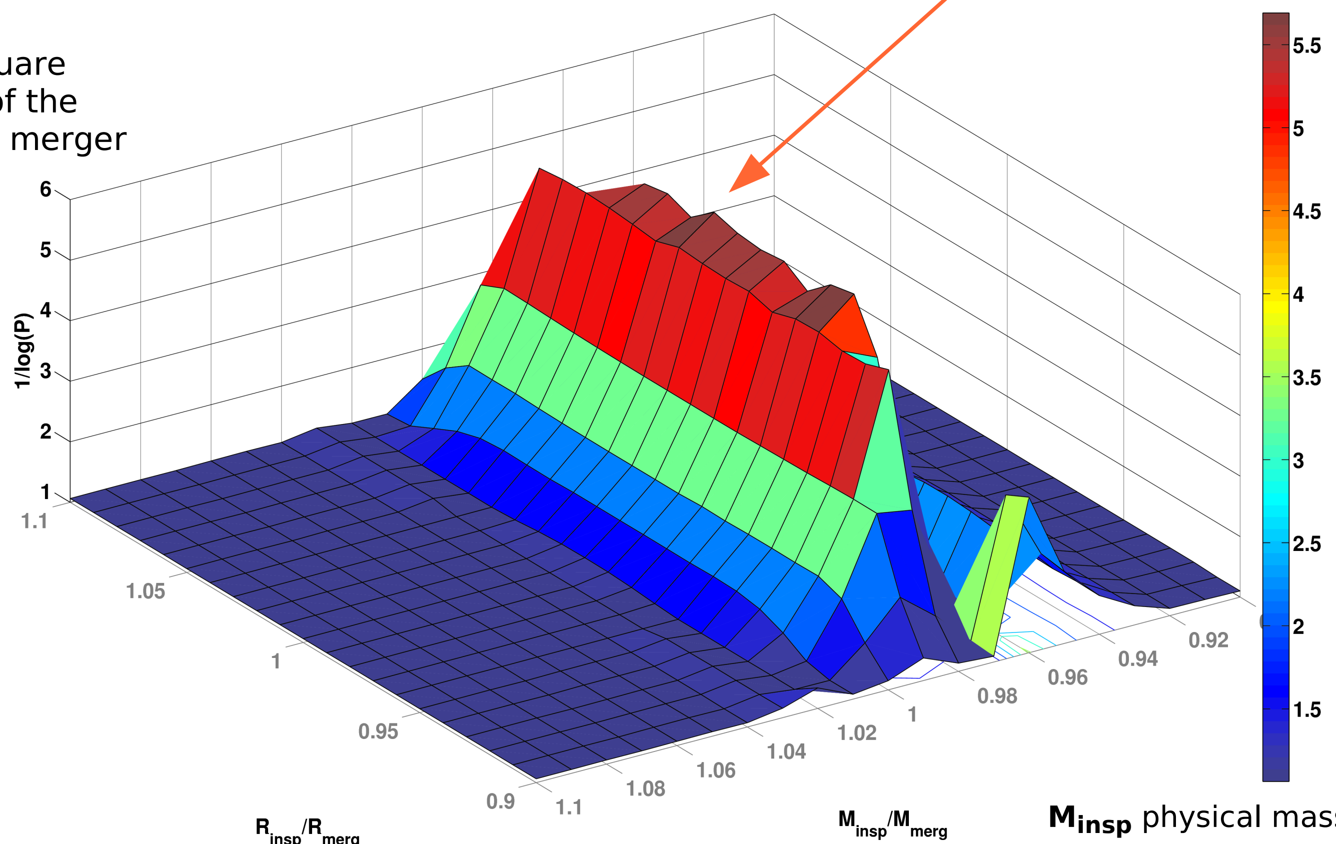
Fourier domain magnitude of the 'hybrid' waveforms constructed using different matching regions.
Merger waveforms used: Equal mass (d5) simulation of the AEI group.
(Similar results by Buonanno, Cook & Pretorius)

Two best fit power-laws ($f^{-7/6}$ and $f^{-2/3}$)

Matching PN and NR waveforms

Best-matched
intrinsic parameters

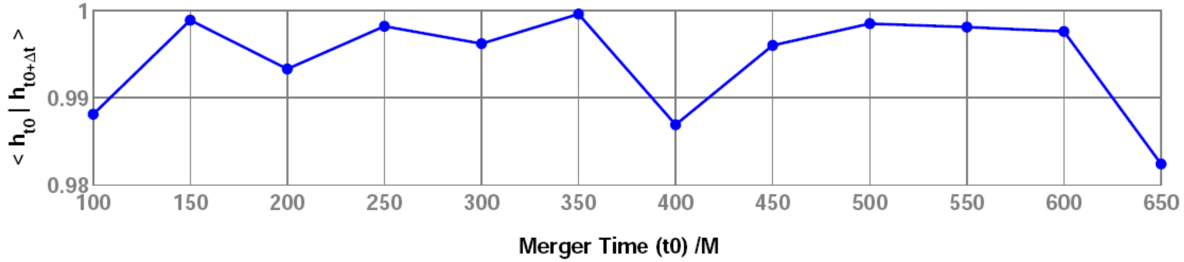
P - least-square
difference of the
inspiral and merger
waveforms



$M_{\mathrm{merg}} = 40\ M_\odot$
$R_{\mathrm{merg}} = 1$
Merger wave: d5 simulation of AEI
Matching region ~ 4 cycles of merger

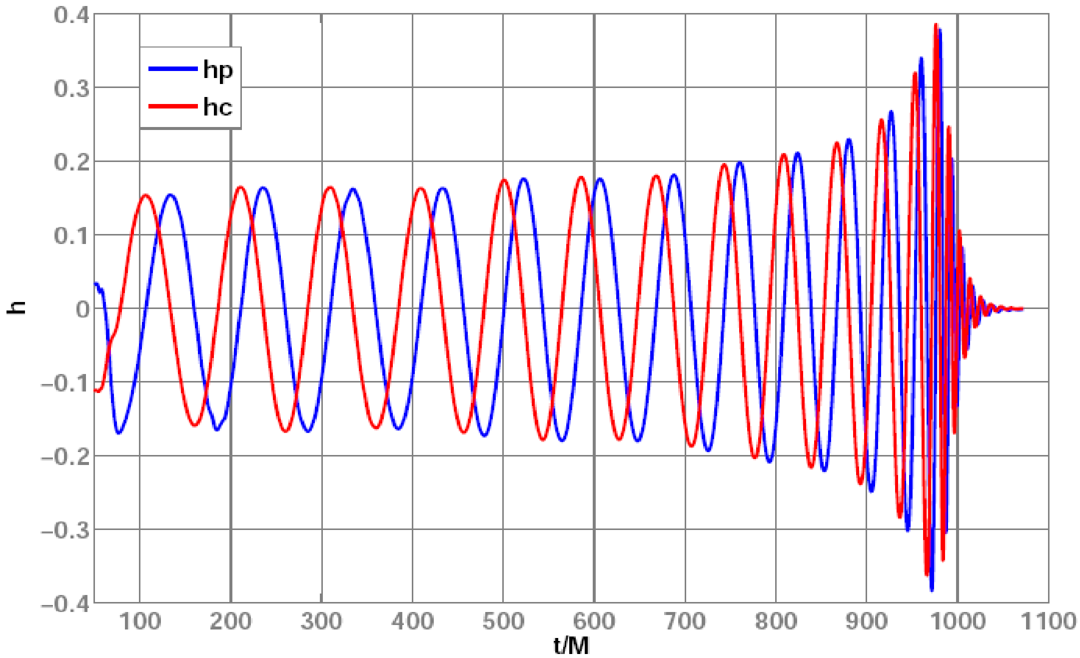
$\mathbf{M_{insp}}$ physical mass of the inspiral
$\mathbf{M_{merg}}$ ADM mass of the merger
$\mathbf{R_{insp}}$ mass-ratio of the inspiral
$\mathbf{R_{merg}}$ mass-ratio of the merger

Consistency of the matching



Overlaps between 'hybrid' waveforms (3.5PN+AEI NR) constructed from a sliding matching region (100 *M* long). Overlaps are calculated using Initial LIGO noise spectrum

- Test the internal consistency of the matching by finding the overlaps between the 'hybrid' waveforms constructed from 'nearby' matching regions (with a sliding matching region) – analogous to a Cauchy convergence test.



Merger waveforms from f500 simulation of the AEI group

Phenomenological waveforms

- Use the following frequency domain parametrisation for the BH coalescence waveforms

$$h(f) \;=\; \mathcal{A}(f)\, \exp\left[i\Psi(f)\right] \,,$$

$$\mathcal{A}(f) \propto \begin{cases} f^{-7/6}\left(1-\alpha^{2/3}\right) & \text{if } f < f_{\rm merg} \\ f^{-2/3} & \text{if } f_{\rm merg} \le f < f_{\rm ring} \\ L(f, f_{\rm ring}, \sigma) & \text{if } f_{\rm ring} \le f < f_{\rm cut} \end{cases}$$

$$\Psi(f) \;=\; f^{-5/6}\left(\psi_0 + \psi_3 f + \psi_4 f^{4/3}\right)$$

Phenomenological waveforms

- Use the following frequency domain parametrisation for the BH coalescence waveforms (motivated by BCV templates)

Transition frequencies

$$h(f) \;=\; \mathcal{A}(f)\,\exp\left[i\Psi(f)\right]\,,$$

Fourier domain waveform

$$\mathcal{A}(f) \propto \begin{cases} f^{-7/6}\left(1-\alpha^{2/3}\right) & \text{if } f < f_{\mathrm{merg}} \\ f^{-2/3} & \text{if } f_{\mathrm{merg}} \le f < f_{\mathrm{ring}} \\ L(f, f_{\mathrm{ring}}, \sigma) & \text{if } f_{\mathrm{ring}} \le f < f_{\mathrm{cut}} \end{cases}$$

Magnitude

$$\Psi(f) \;=\; f^{-5/6}\left(\psi_0 + \psi_3 f + \psi_4 f^{4/3}\right)$$

Cutoff frequency

'Spread' of the Lorentzian

Phase

Lorentzian function

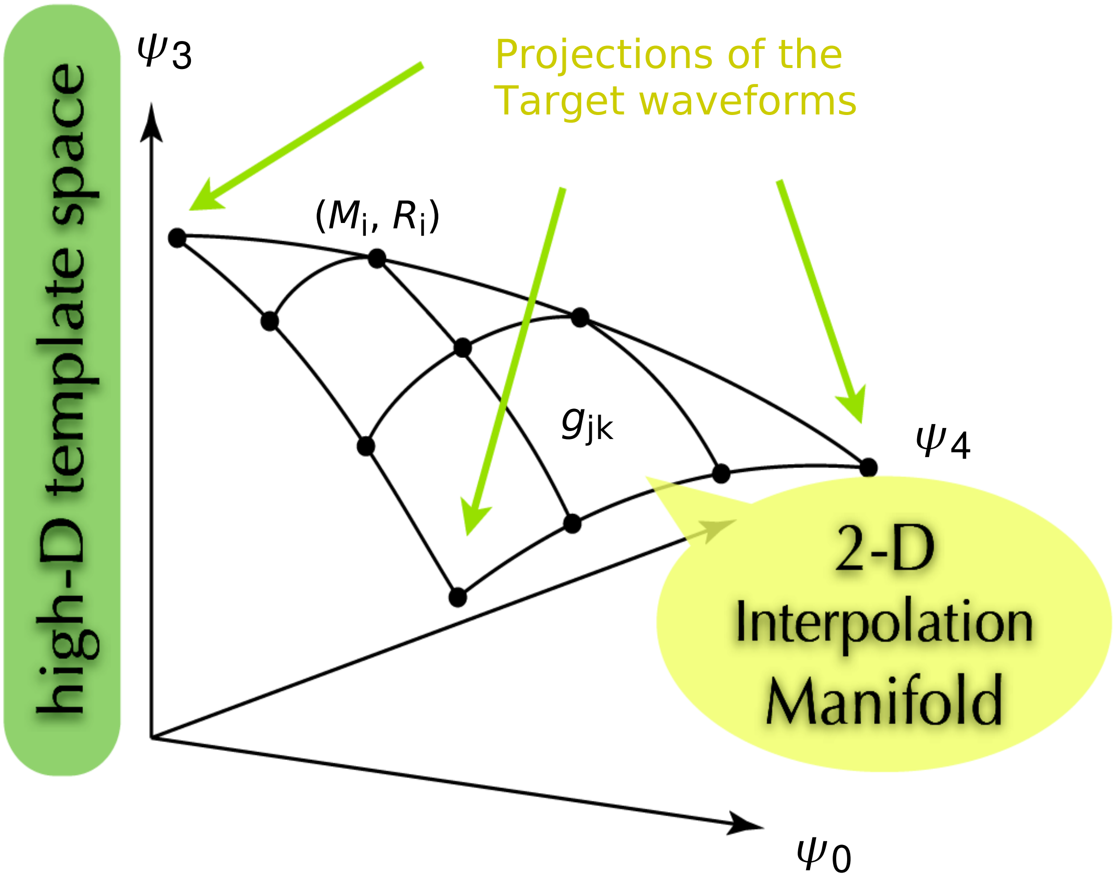
Phase Parameters

Amplitude correction (inspiral)

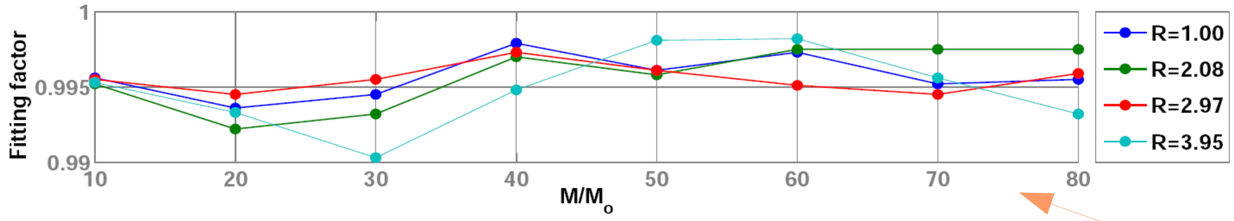
Template bank

- Re-parametrize the templates in terms of *M* and *R*. The template family is a two dimensional (interpolated) manifold embedded in a higher dimensional space, with an induced metric.

- Target waveforms might not be unique for the same physical system. But the parametrisation allows flexibility for incorporating such differences.



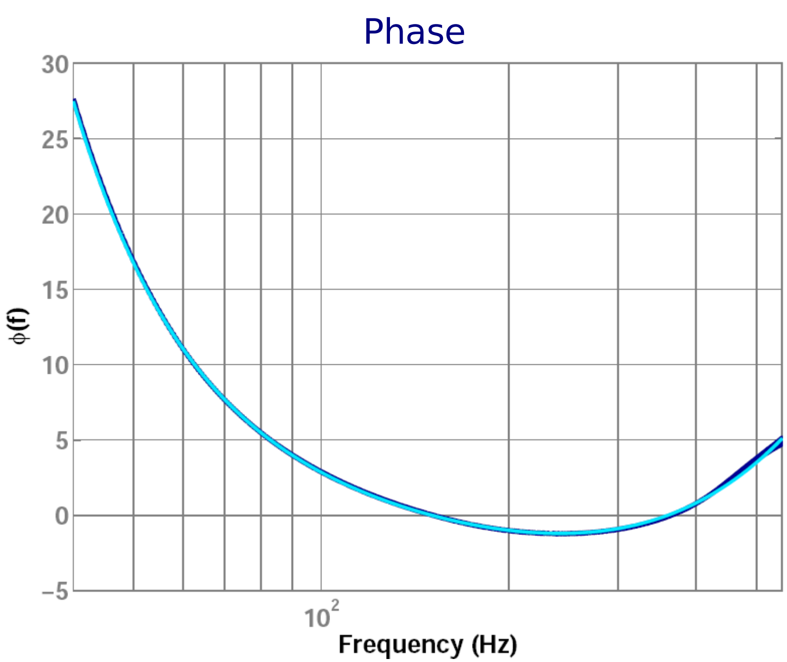
Fitting factors with the target waveforms

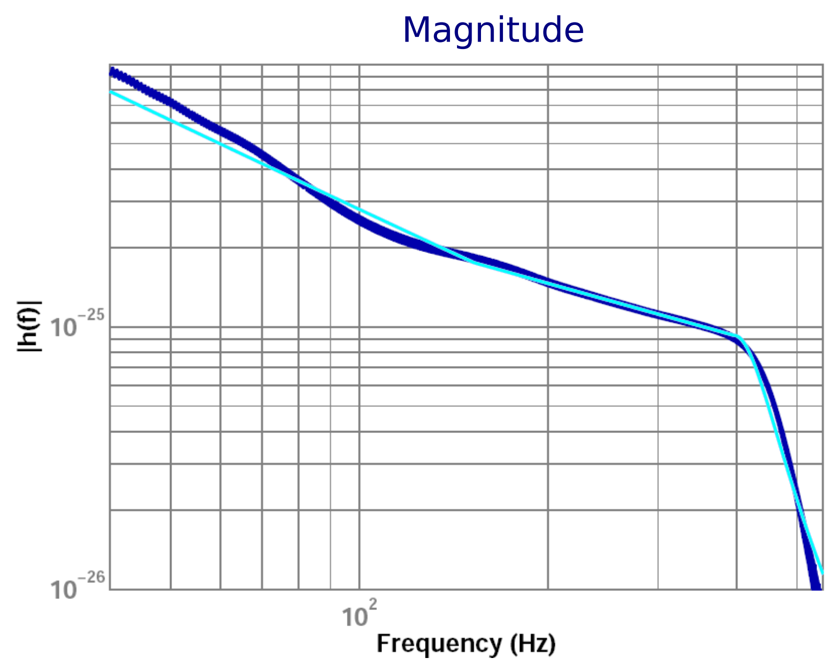


mass-ratio

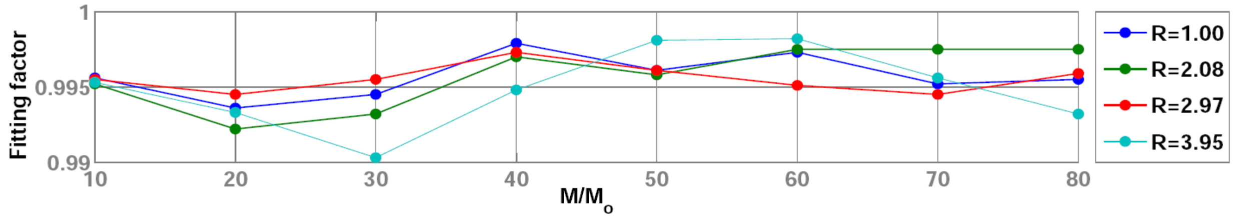
total mass

Fitting factors of the phenomenological templates with the target waveforms (using initial LIGO noise spectrum). Target (hybrid) waveforms are constructed by matching 3.5PN inspiral waveforms with unequal mass merger waveforms produced by the Jena group (1 < mass-ratio < 4).





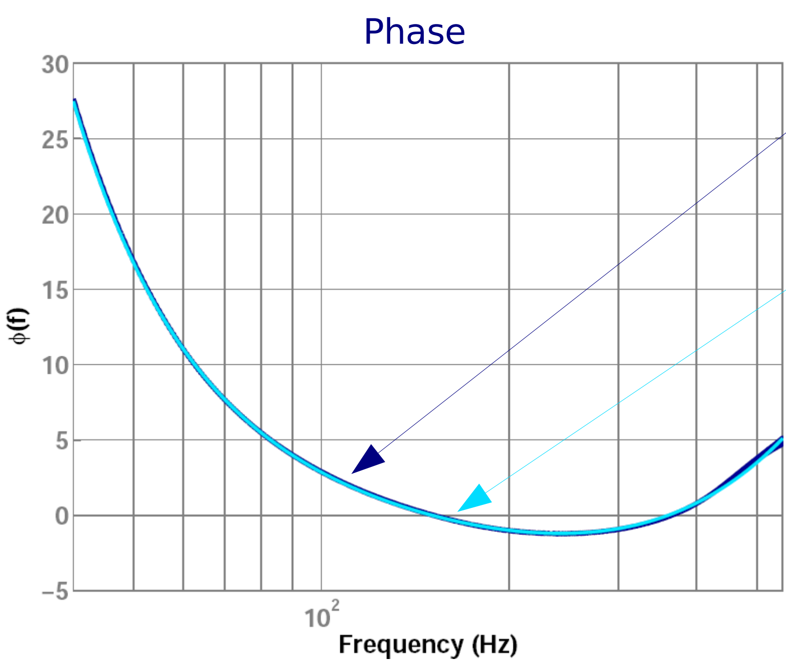
Fitting factors with the target waveforms

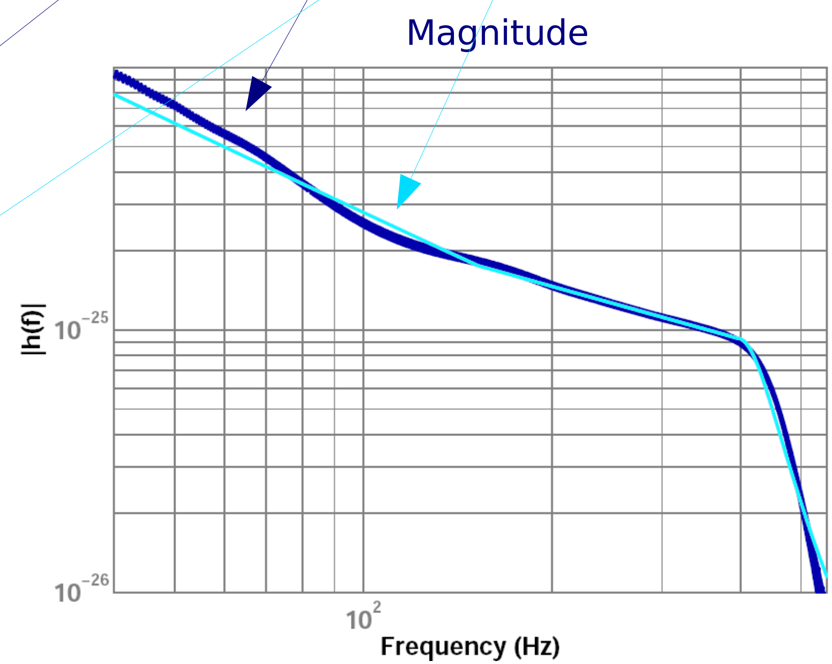


Target waveforms from the
$M = 40\ M_\odot$, $R = 1$ binary
(Jena UM simulation + 3.5PN inspiral)

Fitting factors of the phenomenological templates with the target waveforms (using initial LIGO noise spectrum). Target (hybrid) waveforms are constructed by matching 3.5PN inspiral waveforms with unequal mass merger waveforms produced by the Jena group (1 < mass-ratio < 4).

Best-matched templates

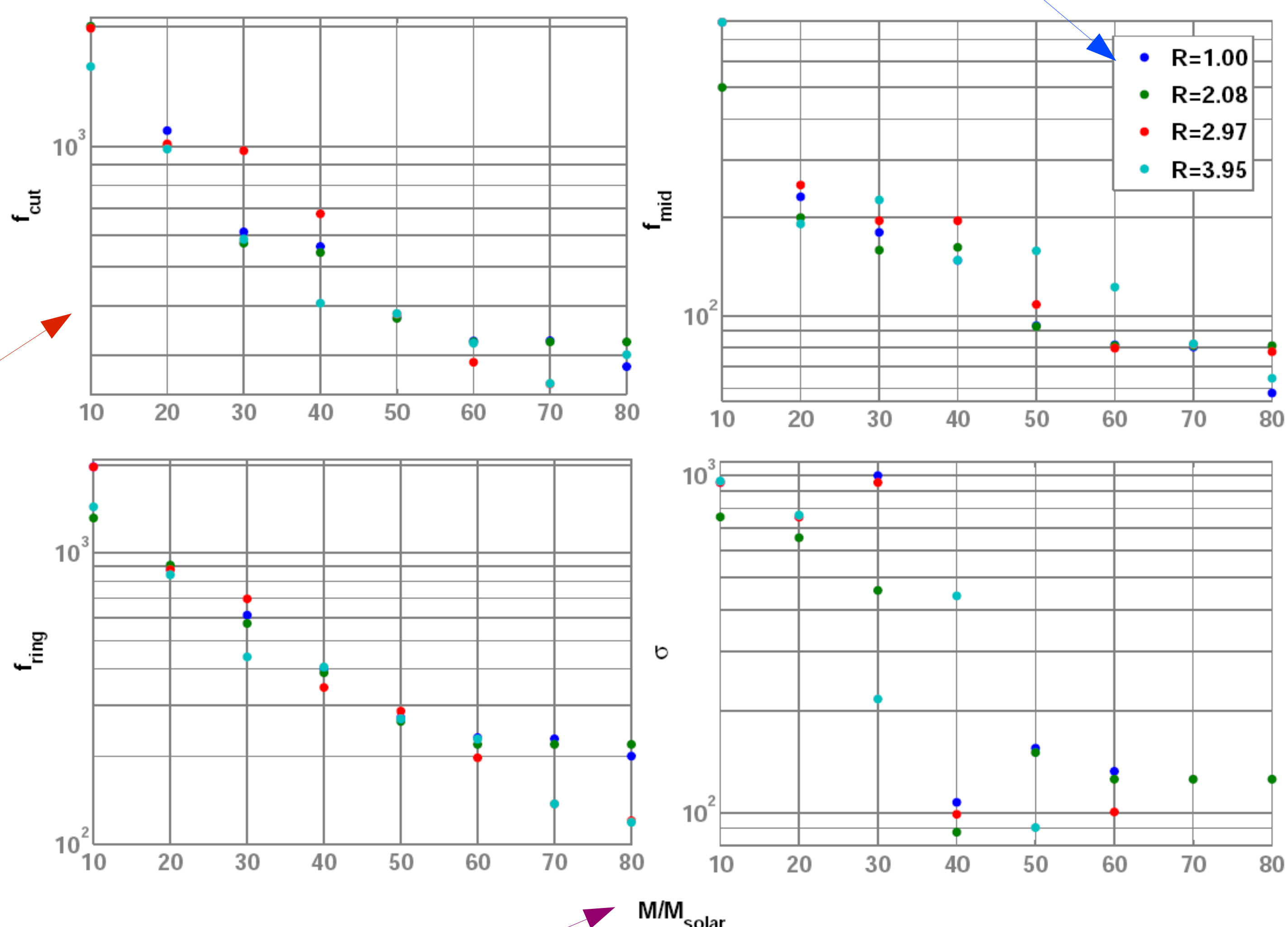




From phenomenological parameters to physical parameters

mass-ratio

Best-matched phenomenological parameters

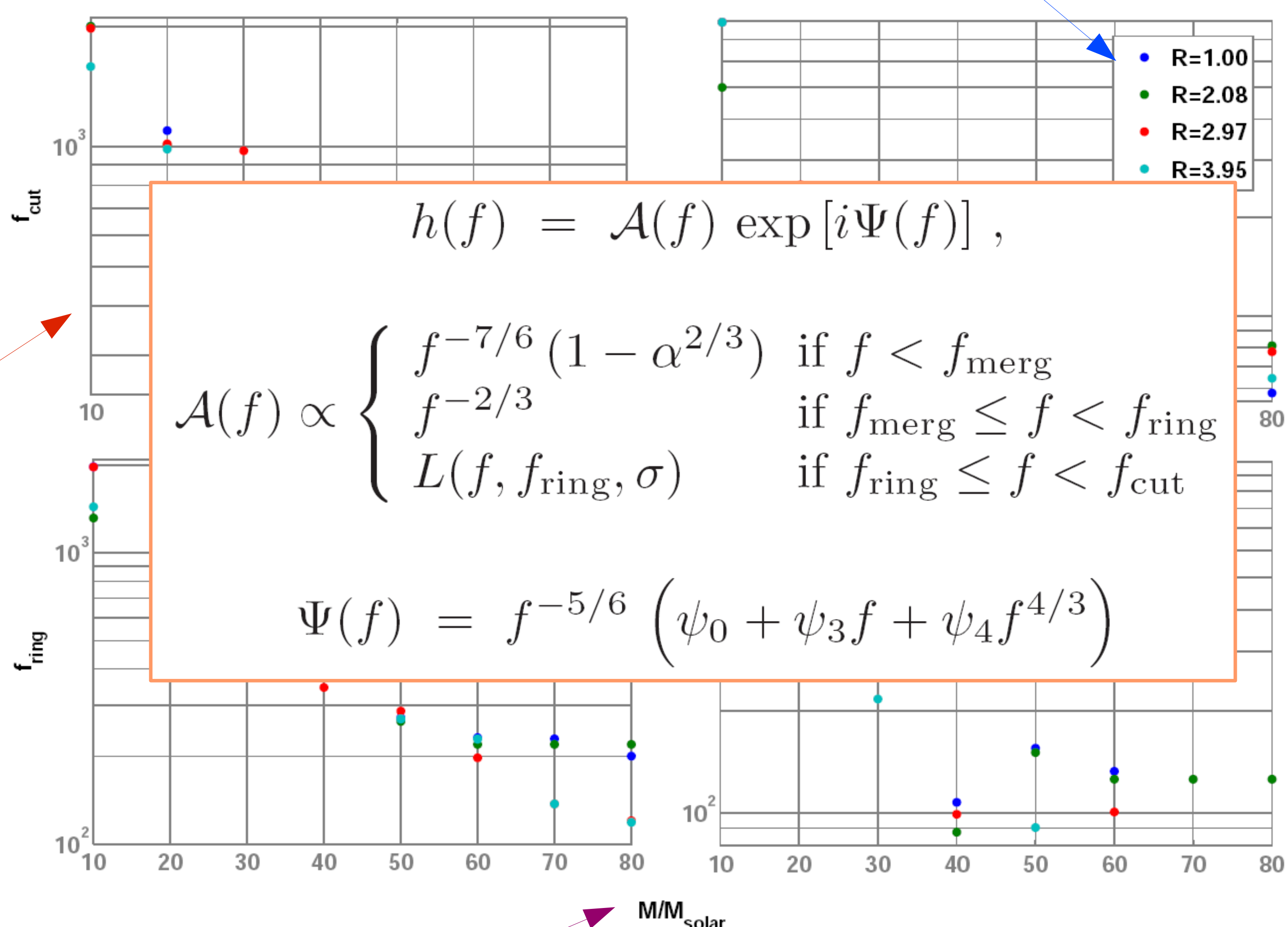


total-mass

Target waveforms: 3.5PN inspiral + Jena UM simulations

From phenomenological parameters to physical parameters

mass-ratio



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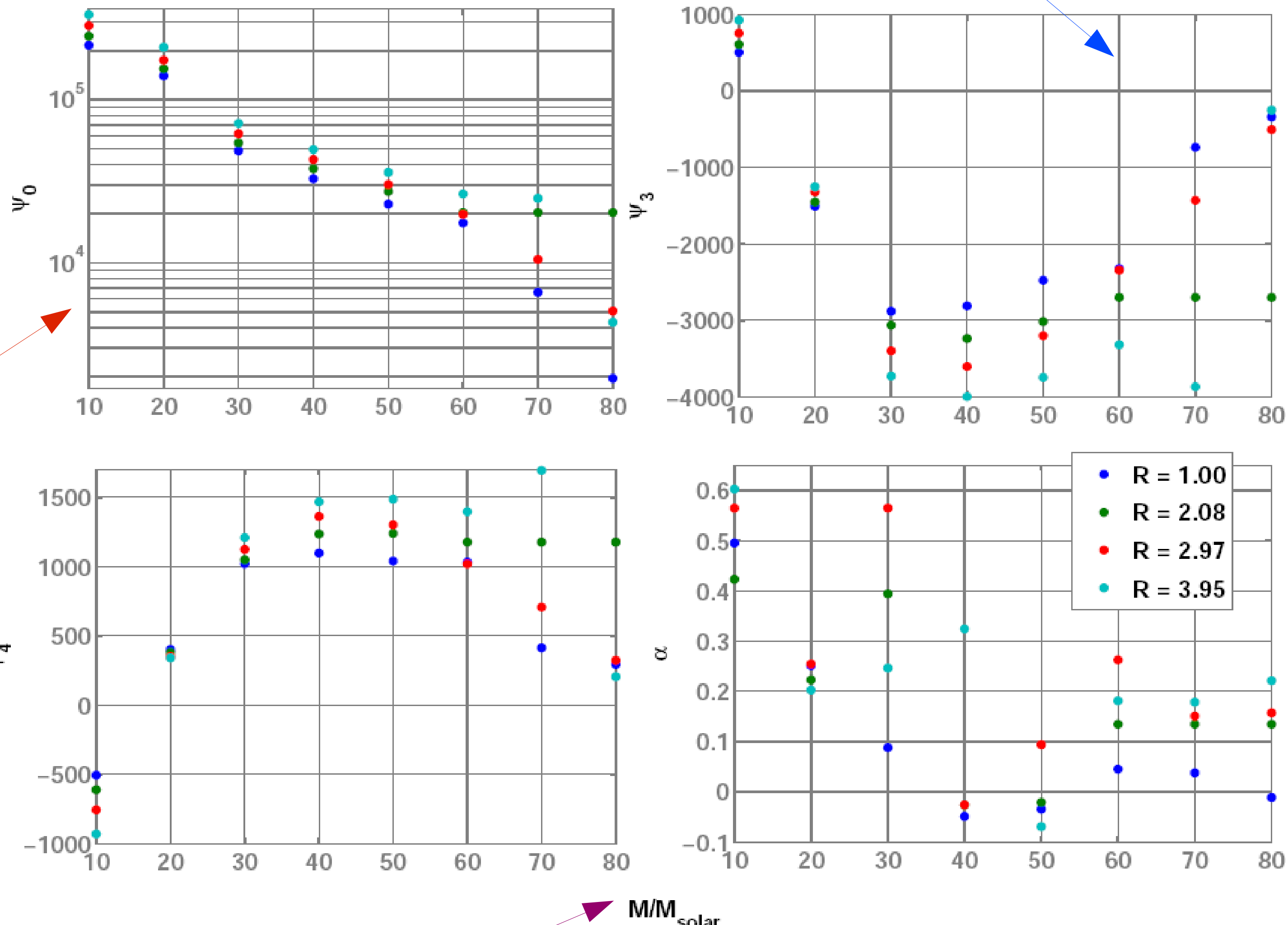
Best-matched phenomenological parameters



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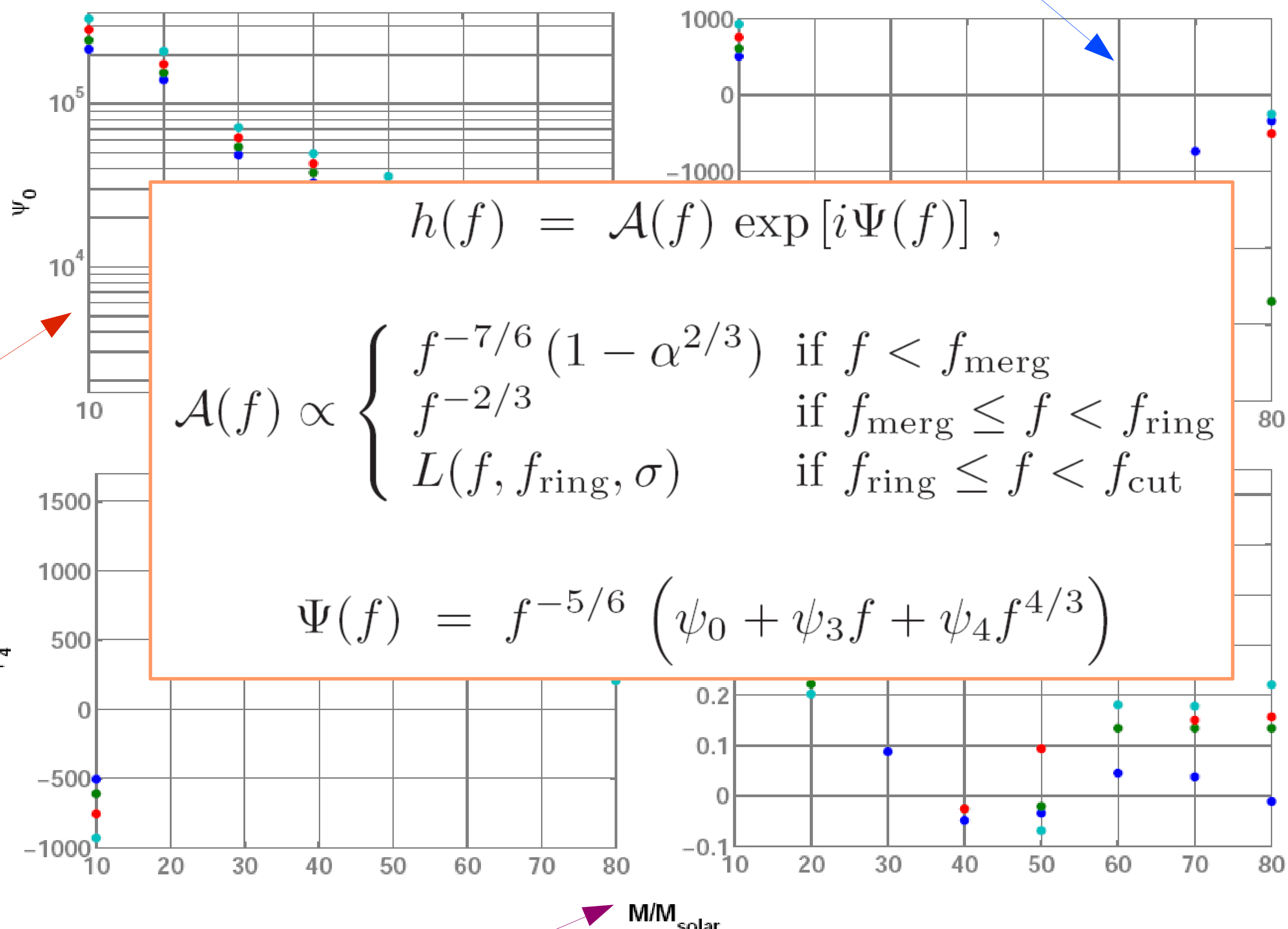

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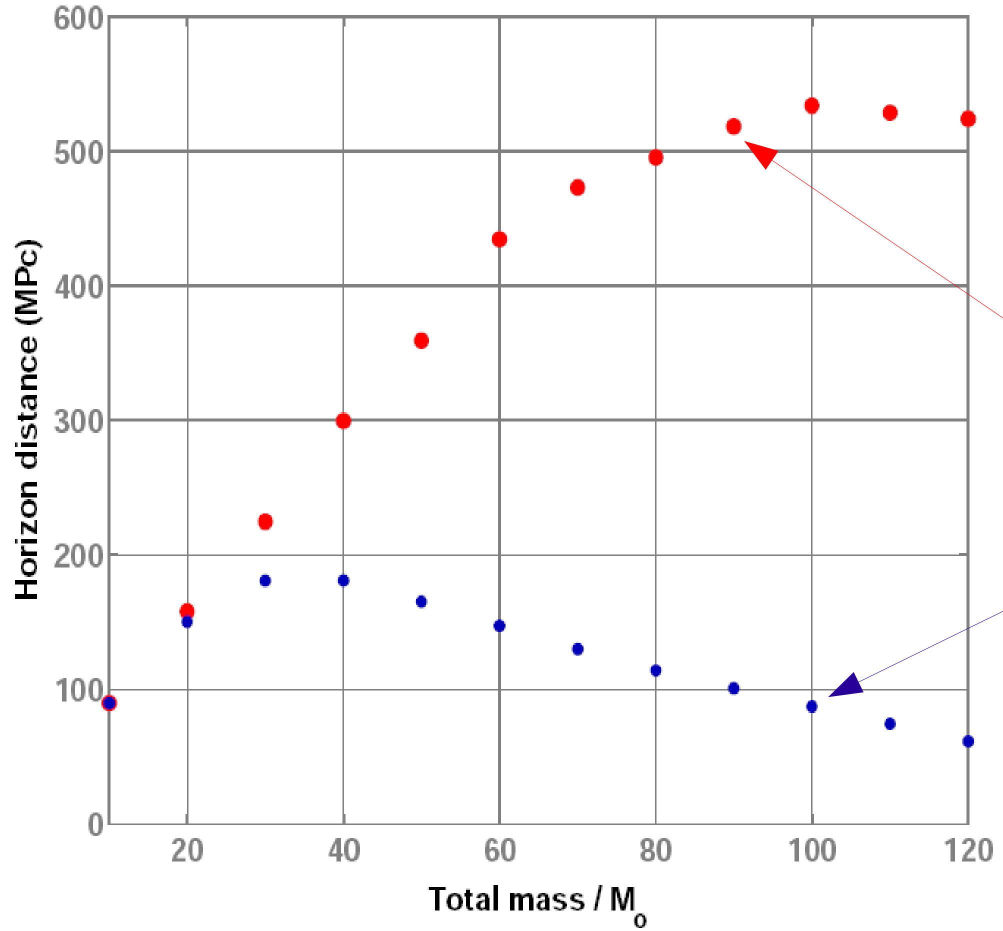
Sensitivity of the search

Distance to the binaries which can produce an optimal SNR of 8 at Initial LIGO



Using inspiral+merger+ring down templates

Using standard PN templates

Summary

- Recent progress in Numerical Relativity in modelling the non-perturbative merger phase of the binary black hole coalescence problem.

- Proposed a phenomenological waveform family which can model the inspiral, merger, and ring-down stages of black hole coalescence (1 < mass-ratio < 4).

- This 8-parameter phenomenological family can be parametrized in terms of two physical parameters → two-parameter template bank.

- This template bank might enable us to extend the present inspiral searches to higher mass binary black hole systems → increased reach of the current generation of ground based detectors.