

An Evidence Based Search For Neutron Star Ringdowns

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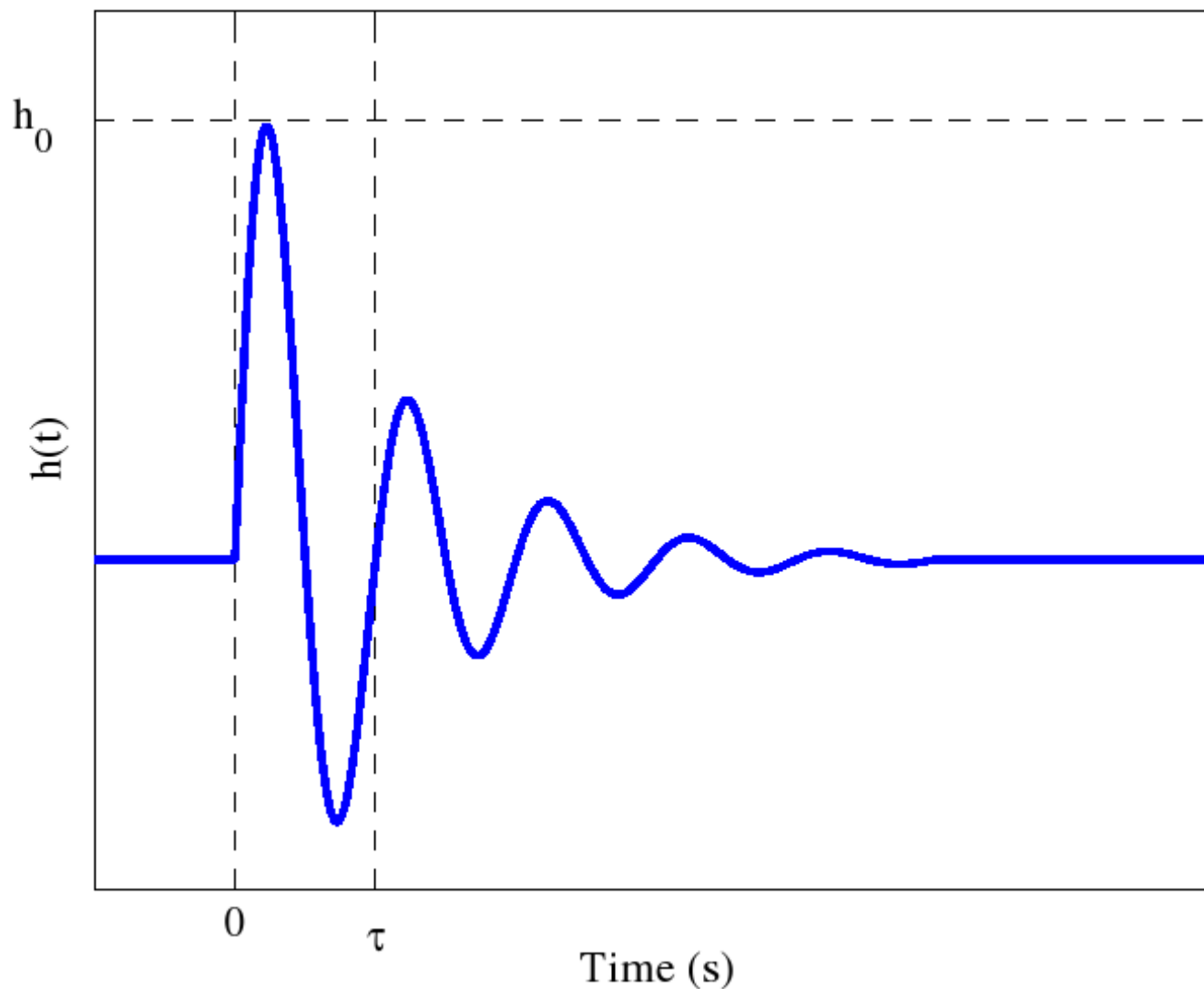
Objective: Construct a (triggered) Bayesian search algorithm for neutron star ring-downs

- Neutron star ring-downs
- Bayesian model selection & evidence
- Application & analysis pipeline
- Preliminary sensitivity estimates
- Future work

Possible GW emission from neutron stars via quasi-normal mode (QNM) oscillations. QNMs may be excited by (e.g.):

- Birth of neutron star in core-collapse supernova
- **Soft gamma repeater (SGR) flares:**
 - highly magnetised NS, B-field stresses induce crustal cracking & excite QNMs, leading to GWs
 - Trigger: GRB observations (e.g., SGR1806-20 – GEO & LHO data)
- **Pulsar glitches**
 - Spin-down (and or de-coupling of crust/core, internal phase transition) induces crustal cracking due to relaxation of ellipticity: starquake.
 - Trigger: pulsar timing data

$$h_0 \sim 5 \times 10^{-24} \left(\frac{E}{10^{-11} M_{\odot} c^2} \right)^{1/2} \left(\frac{\tau}{200 \text{ ms}} \right)^{-1/2} \left(\frac{\nu}{2 \text{ kHz}} \right) \left(\frac{D}{15 \text{ kpc}} \right)^{-1}$$



- For competing models $\mathcal{M}_1, \mathcal{M}_2$ compute the **odds ratio** (ratio of posteriors probabilities) :

$$O_{12} = \frac{p(\mathcal{M}_1 | \{\mathcal{D}\}, I)}{p(\mathcal{M}_2 | \{\mathcal{D}\}, I)}$$

- Odds ratio consists of 2 terms:

$$O_{12} = \underbrace{\frac{p(\mathcal{M}_1 | I)}{p(\mathcal{M}_2 | I)}}_{\text{prior odds}} \times \underbrace{\frac{p(\{\mathcal{D}\} | \mathcal{M}_1, I)}{p(\{\mathcal{D}\} | \mathcal{M}_2, I)}}_{\text{Bayes factor}}$$

- $p(\{\mathcal{D}\} | \mathcal{M}_i, I)$ is the **evidence** for the model (likelihood, marginalised over some model parameters $\underline{\theta}$ and weighted by the prior) :

$$p(\{\mathcal{D}\} | \mathcal{M}_i, I) = \int_{\underline{\theta}} p(\underline{\theta} | \mathcal{M}_i, I) \times p(\{\mathcal{D}\} | \underline{\theta}, \mathcal{M}_i, I) d\underline{\theta}$$

\mathcal{M}_1 : Data contains a ring-down in Gaussian white noise

- Likelihood function for a single datum \mathcal{D} , given an arbitrary signal power S & Gaussian noise σ is a non-central chi-squared distribution with non-centrality parameter S :

$$p(\mathcal{D} | S, \sigma, M) = \frac{1}{2\sigma^2} \exp\left[-\frac{\mathcal{D} + S}{2\sigma^2}\right] I_0\left(\frac{\sqrt{\mathcal{D}S}}{\sigma^2}\right)$$

- where S is modelled with a Lorentzian line profile, parameterised by

$$\underline{\theta} = \{h_0, \nu_0, \tau\}$$

 \mathcal{M}_2 : Data only contains Gaussian white noise

- Know *a priori* that the 'signal power' is zero – use the same likelihood function with a strong prior on S to get the central chi-squared distribution for the evidence

$$p(S | \mathcal{M}_2) = \delta(S)$$

Choice of priors for \mathcal{M}_1

- Assume parameters are independent so that:

$$p(h_0, \nu_0, \tau) = p(h_0)p(\nu_0)p(\tau)$$

$$e.g., p(h_0) = \frac{1}{\max(h_0) - \min(h_0)}$$

Parameter	Prior	Range ^a
amplitude, h_0	uniform	0 – 5×10^{-20}
frequency, ν_0	uniform	(1500 – 3000) Hz
decay time, τ	uniform	0.05 – 0.5 s

^azero outside of this range

Aim is to detect a known waveform in a stretch of noisy interferometer data with known properties:

- $p(\mathcal{M}_1 | \{\mathcal{D}\}, I)$ - probability that the data contains a ring-down waveform and white noise
- $p(\mathcal{M}_2 | \{\mathcal{D}\}, I)$ - probability that the data contains only white noise

→ Odds ratio O_{12} acts like a detection statistic for ring-downs versus white noise

1. Construct spectrogram centered on external trigger (e.g., pulsar glitch)

2. Compute all possible \mathcal{M}_1 & \mathcal{M}_2 likelihoods for pixels & marginalise to get evidences in each time bin

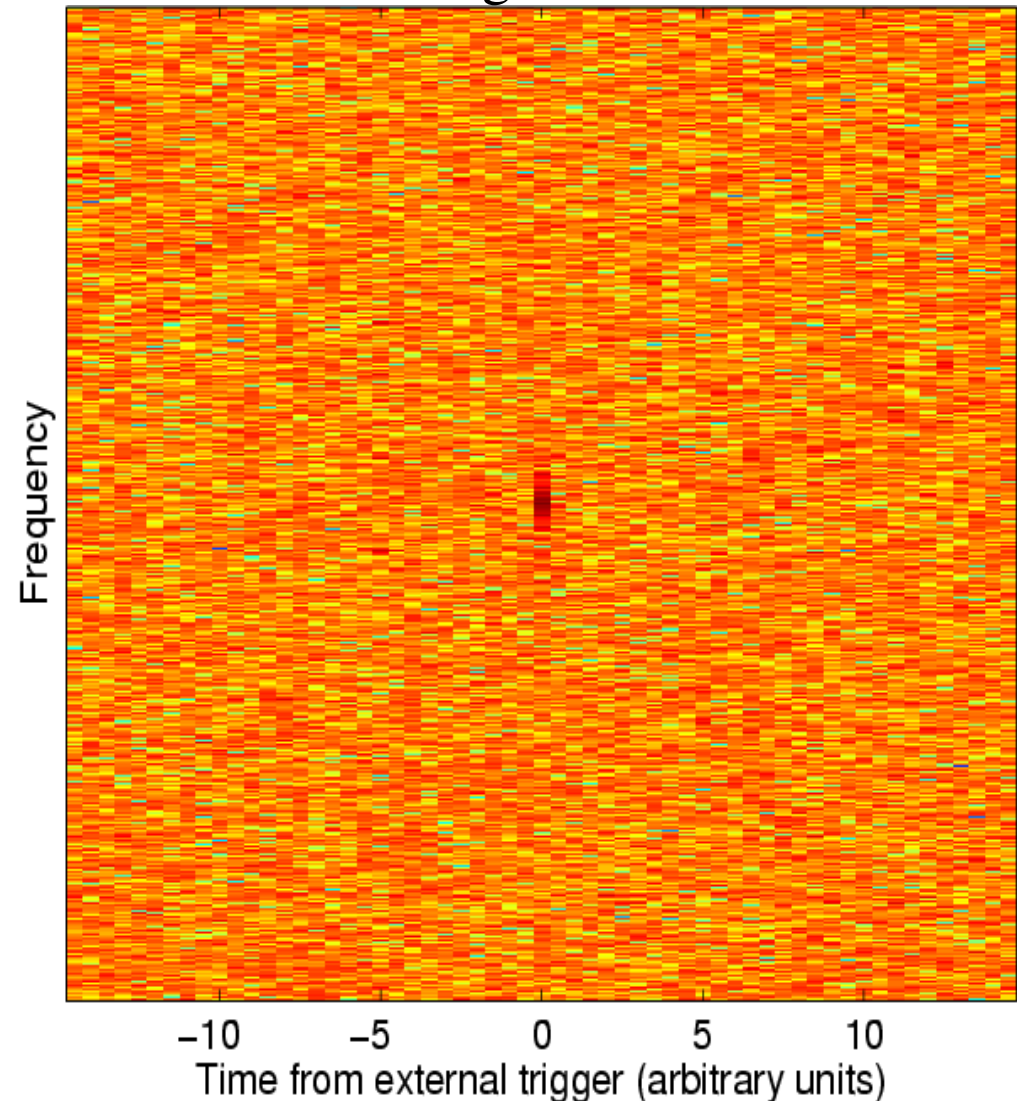
3. Assume no prior model bias and compute odds ratio:

$$4. \quad O_{12} = \frac{p(\{D\} | \mathcal{M}_1, I)}{p(\{D\} | \mathcal{M}_2, I)}$$

5. Finally, identify events with:

$$O_{12} > O_{thresh}$$

illustrative example spectrogram with ringdown:



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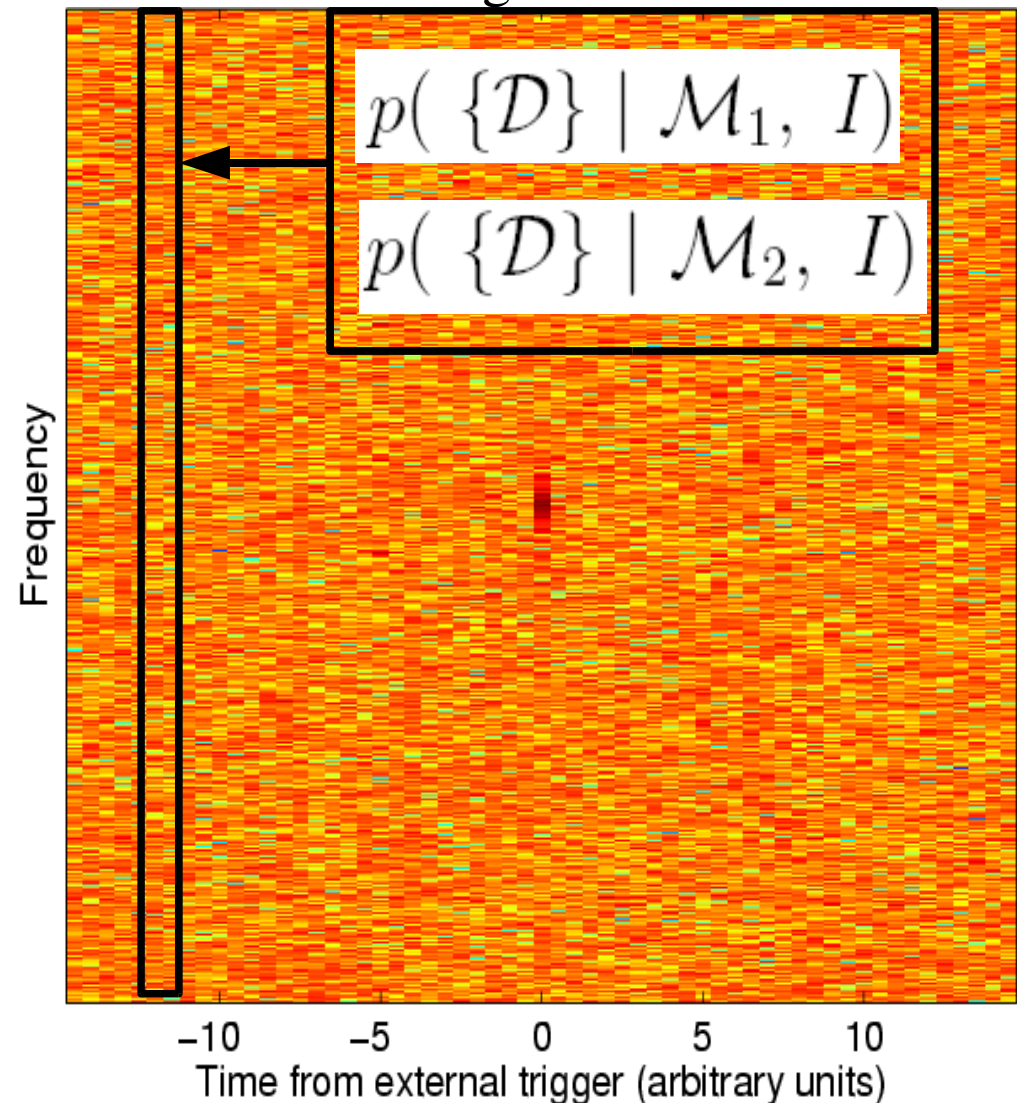
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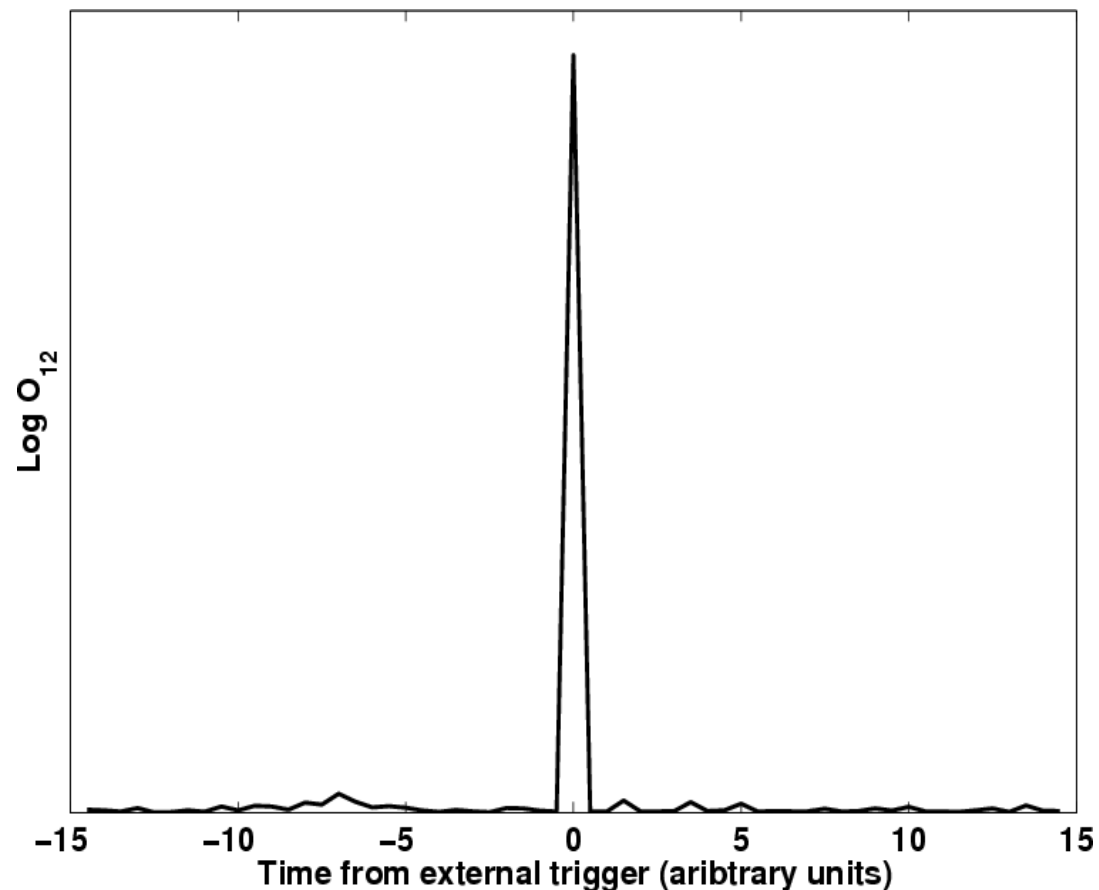
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log odds from previous example:



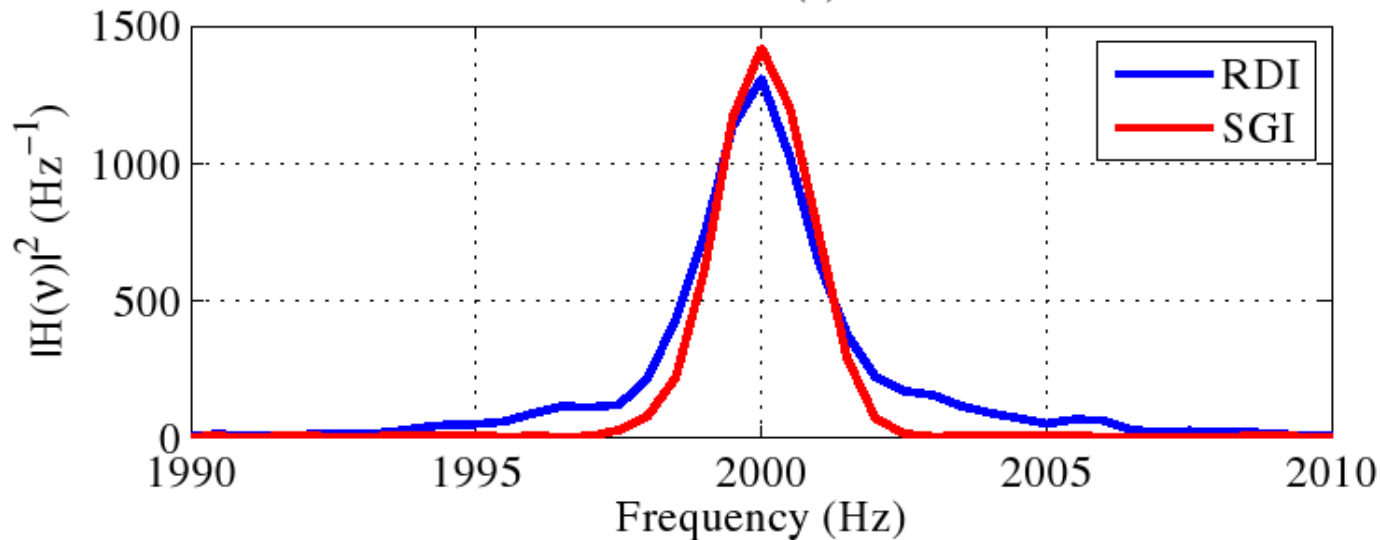
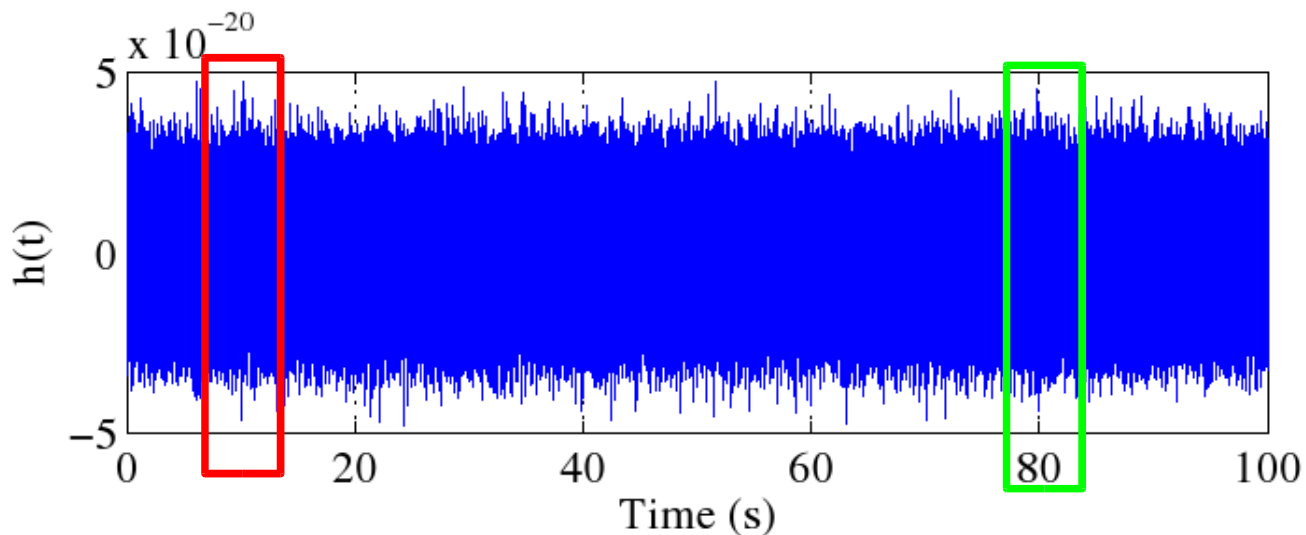
An Example

- Compare response to target (ring-down - RD) waveform and an unwanted glitch (sine-Gaussian - SG)

RD

SG

Inject 1 ring-down and 1 sine-Gaussian of roughly equal SNR into synthetic Gaussian white noise



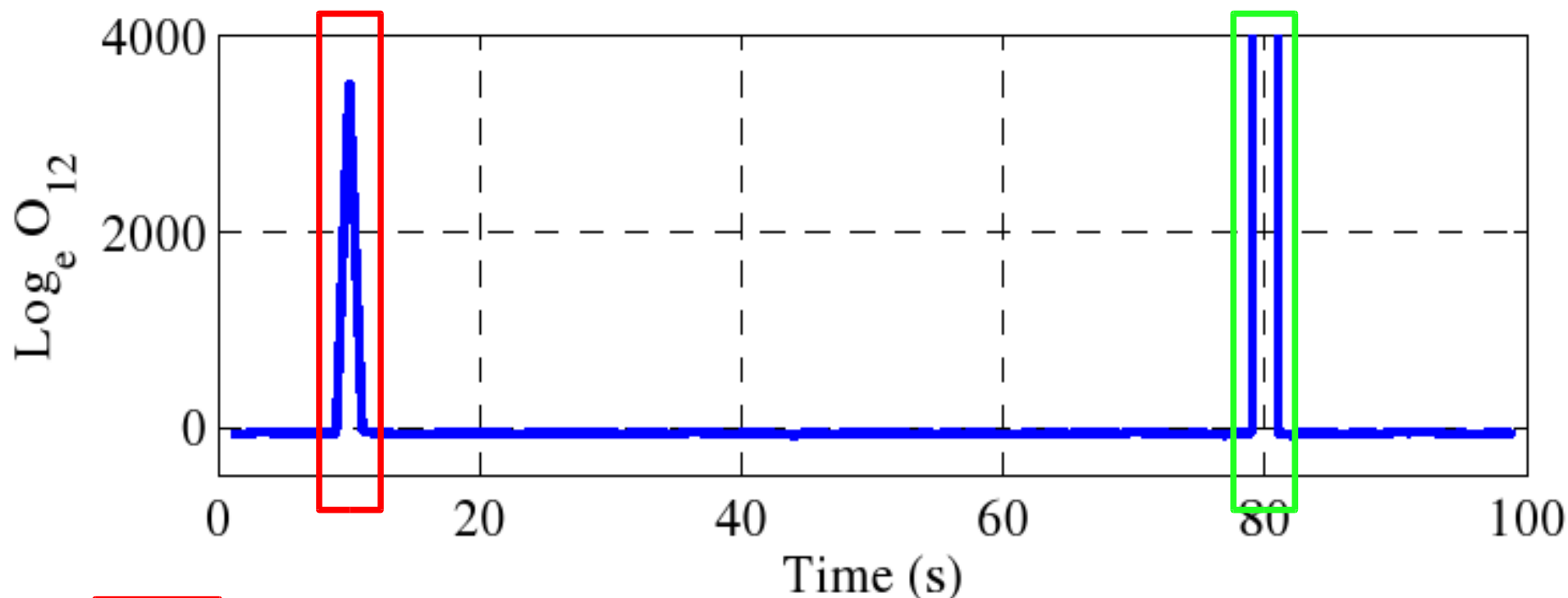
$$\rho_{RD} \approx \rho_{SG} \approx 40$$

$$\rho = \frac{h_{r_{SS}}}{ASD_{noise}}$$

$$h_{r_{SS}} = \sqrt{\int_{-\infty}^{+\infty} |h(t)|^2 dt}$$

Example Output

Output from odds algorithm:



- **RD** : ring-down is detected with odds well above that of background
- **SG** : sine-Gaussian is **also** detected

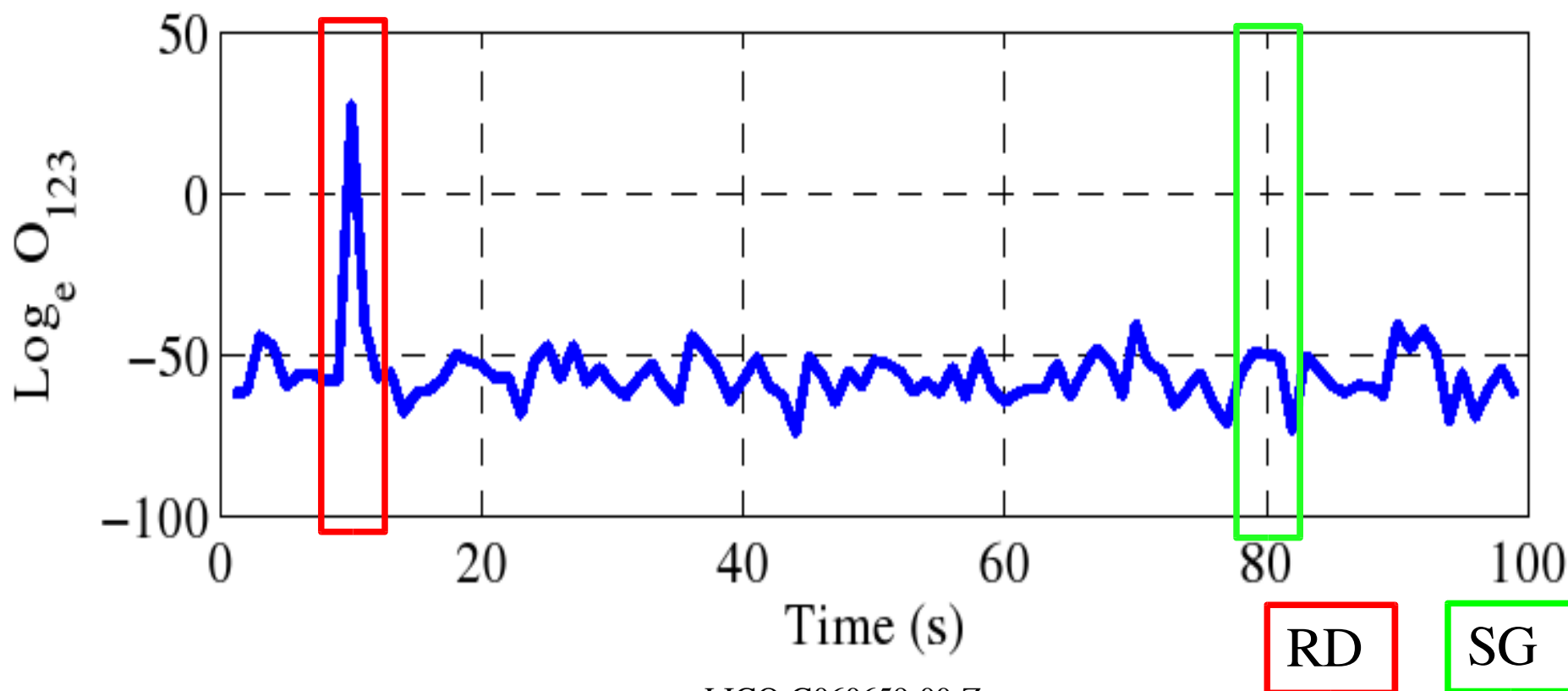
In fact, for the sine-Gaussian: $O_{12} \rightarrow +\infty$ as $p(M_2|D, I) \rightarrow 0$

Example Output

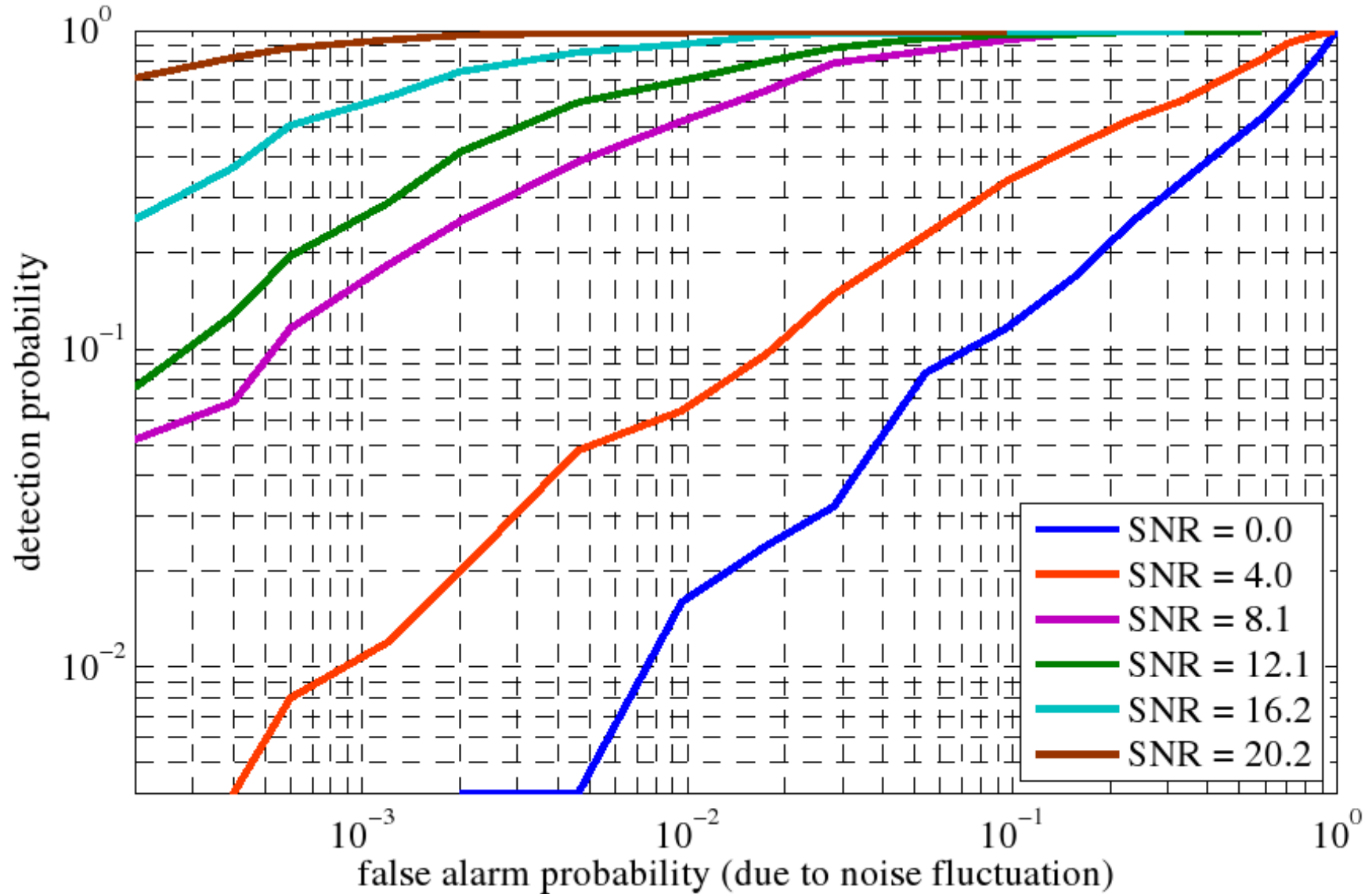
Solution - consider an alternative 'glitch' hypothesis

\mathcal{M}_3 : data contains a sine-Gaussian in Gaussian white noise

$$O_{123} = \frac{p(\mathcal{M}_1 | \mathcal{D}, I)}{p(\mathcal{M}_2 | \mathcal{D}, I) + p(\mathcal{M}_3 | \mathcal{D}, I)}$$



Receiver operating characteristics



Short-term:

- Finish writing up methodology (J. Clark et al. in preparation)
- Run code on GEO & LIGO data from around SGR1806-20 – need to know what happens with *real* data... (have data)

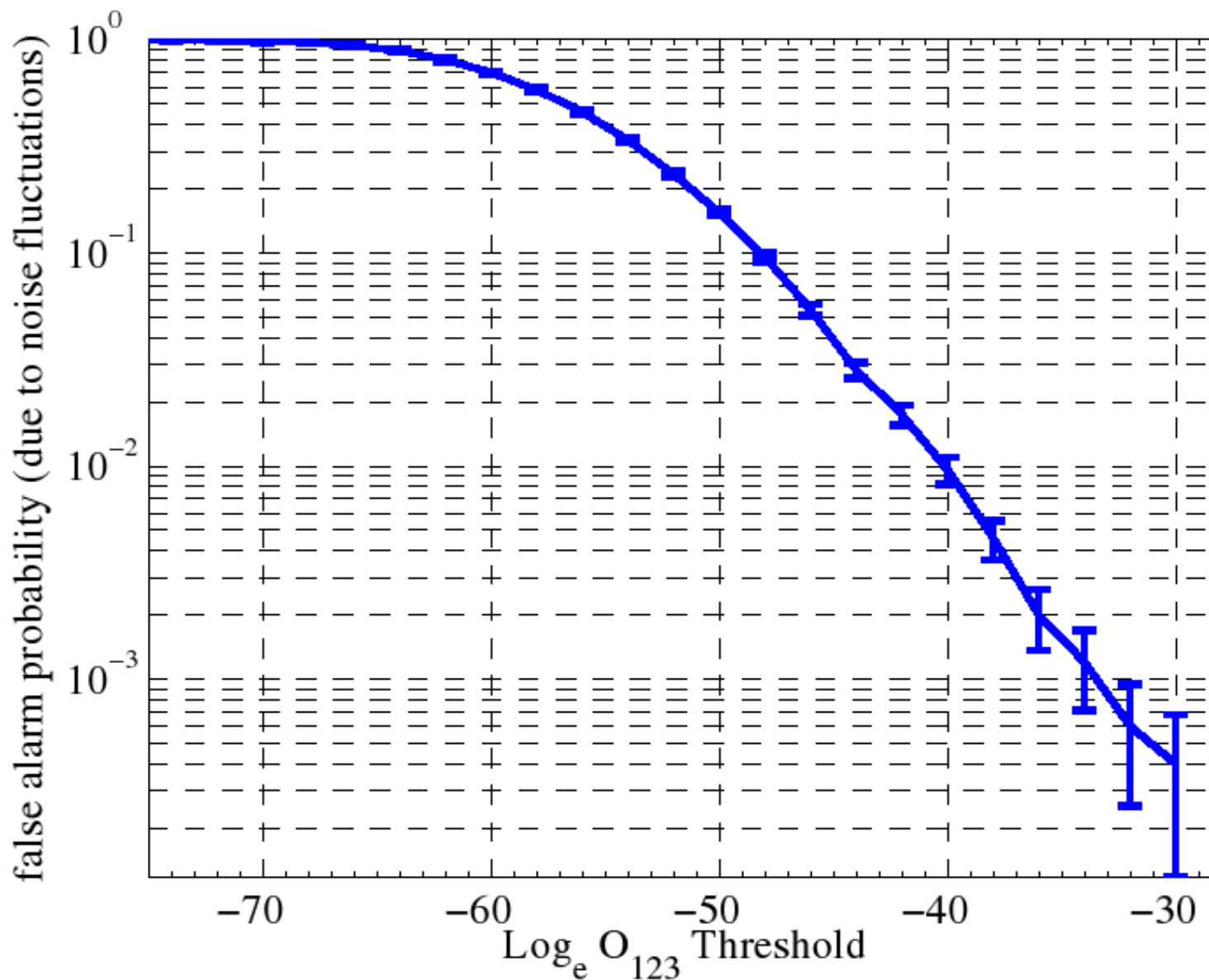
Long-term:

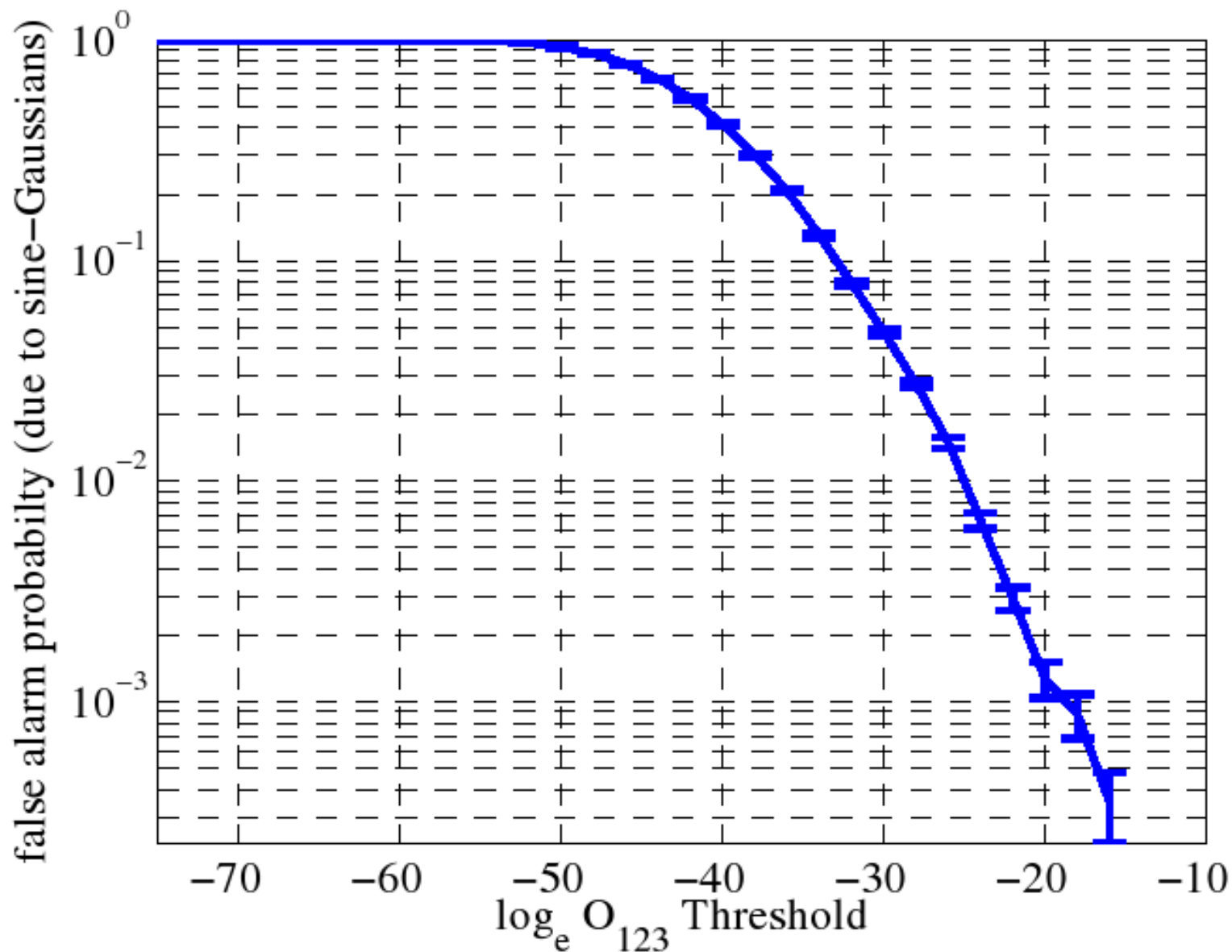
- Upper limits on SGR1806-20 based on posterior probabilities and/or search sensitivity
- Look at other sources (pulsar glitches, GRB ring-downs)
- Potentially have a framework for multi-detector analysis from joint probabilities between detectors



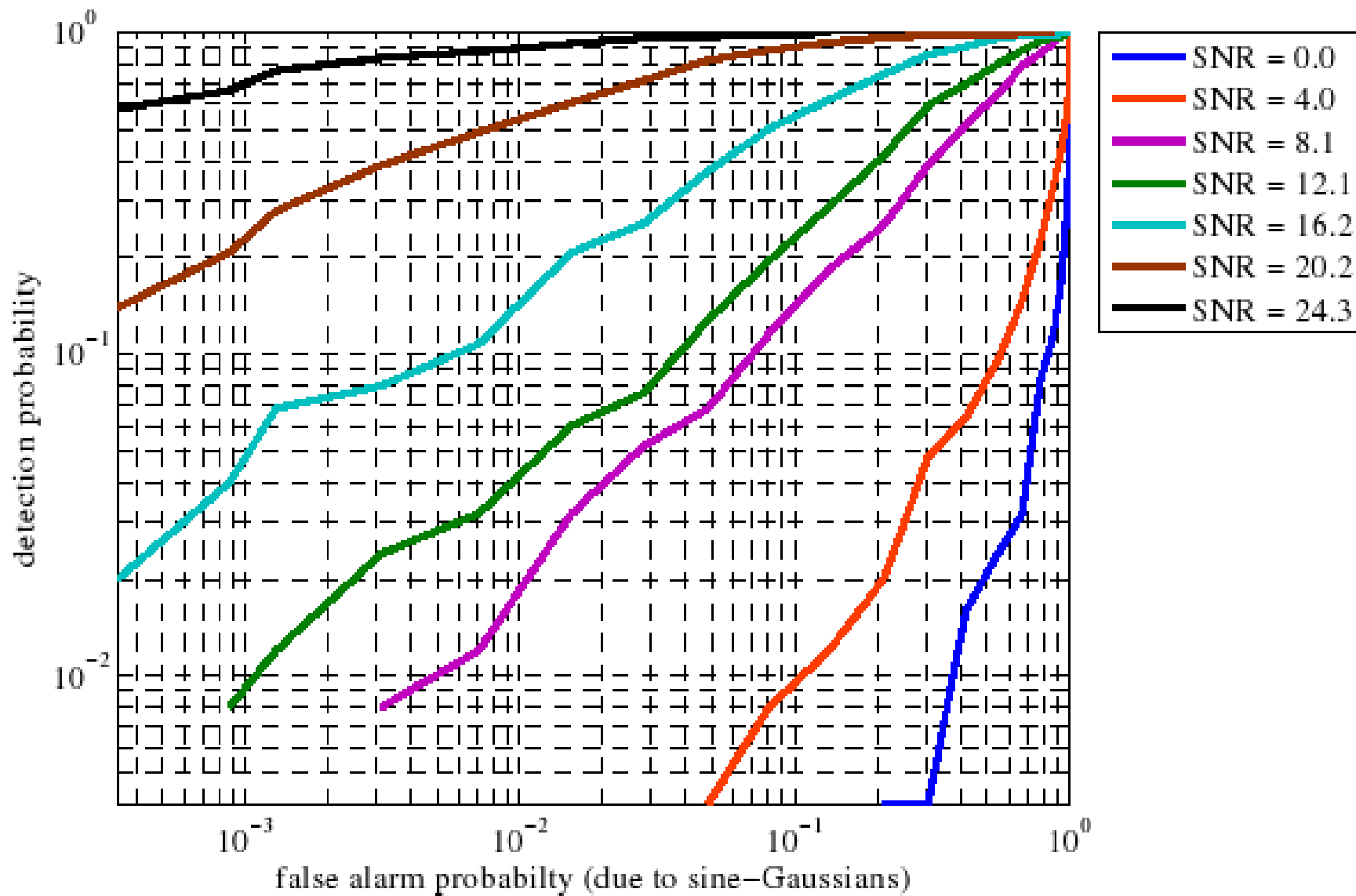
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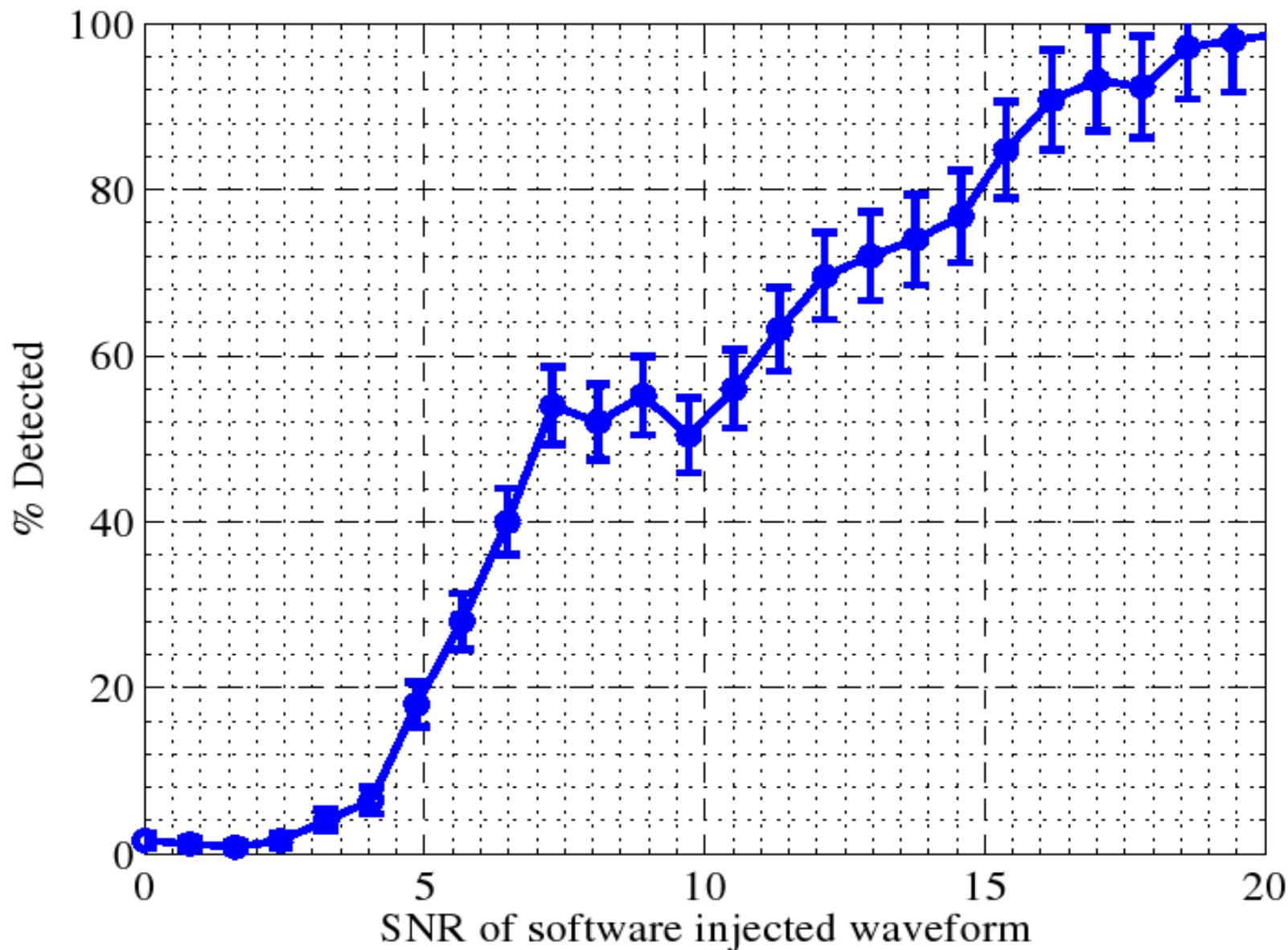


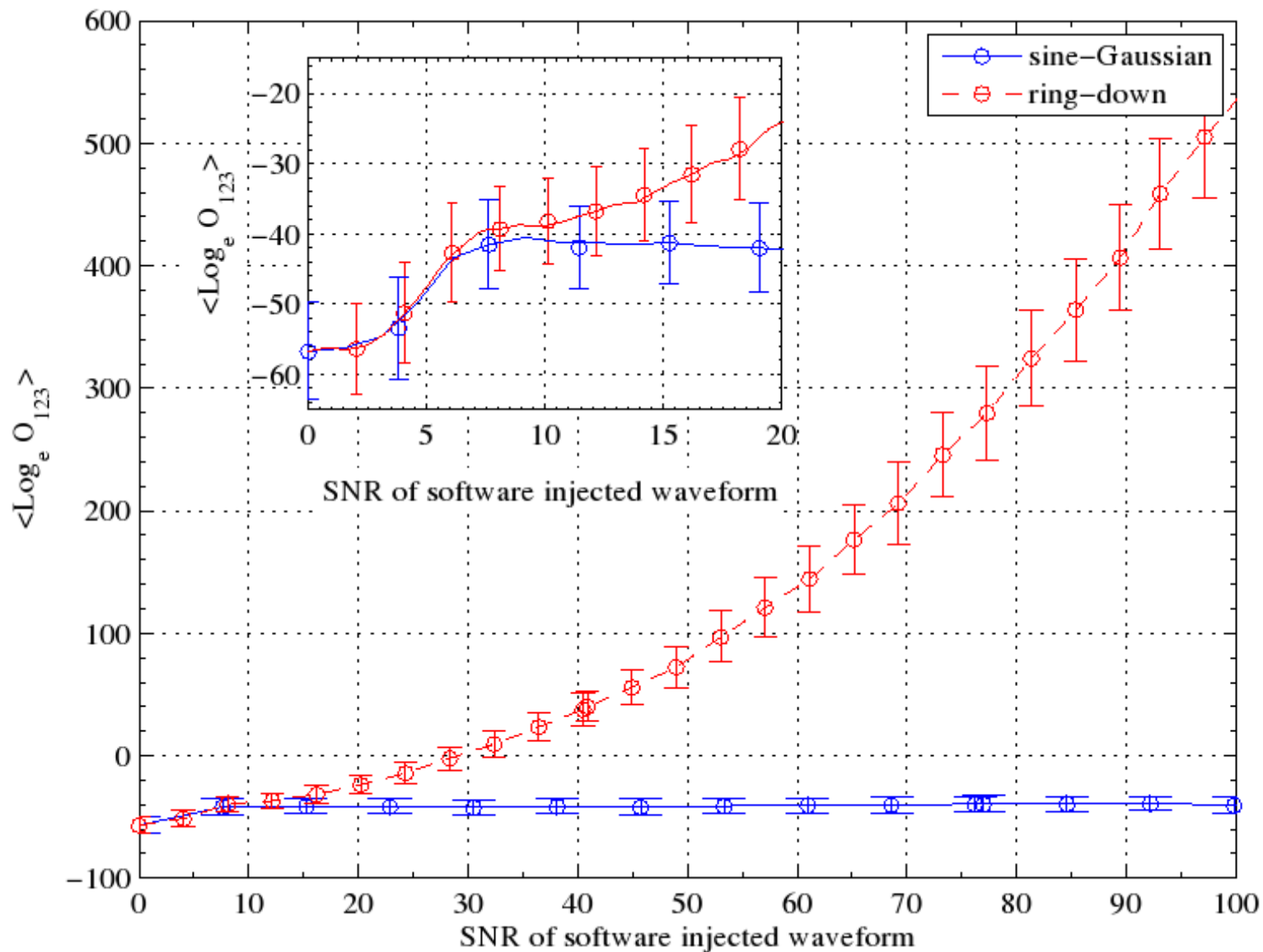




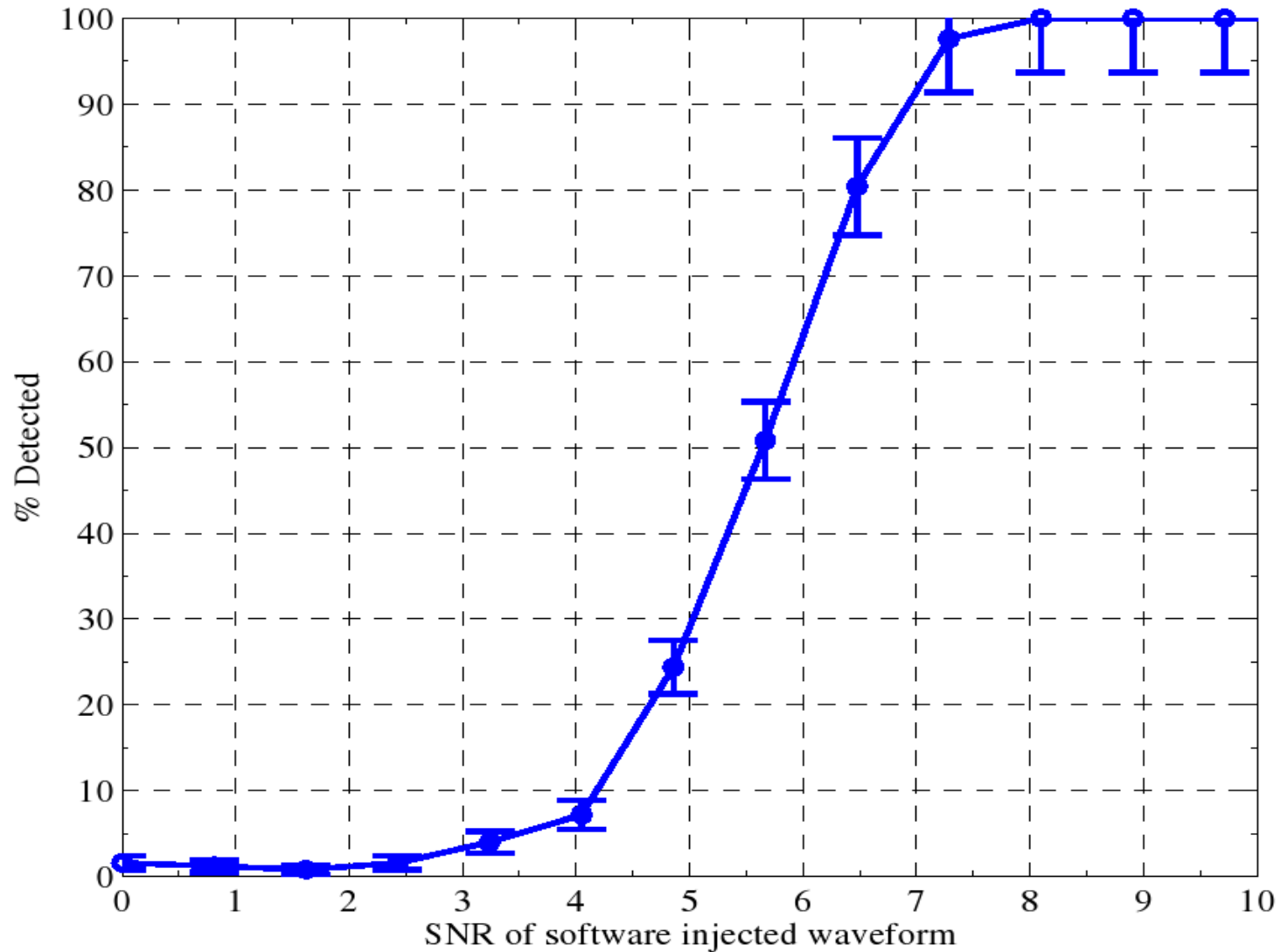
Sensitivity Estimates







Using original odds ratio, O_{12} :



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