A "fast" parameter space search for continuous gravitational waves from known binary systems

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GWDAW 11

Outline

Introduction

- Non-axisymmetric spinning neutron stars are thought to be candidates for continuous gravitational waves sources.
- "Targetted" searches for known isolated and binary pulsars have been/are being done at an unprecented sensitivity.
- "Parameter space" searches fall into 2 catagories
  - "blind" searches eg. all sky search [Col06]
  - "semi-targetted" searches where the parameter space is constrained by previous EM observations eg. Sco X-1 search [Col06]
- Parameter space searches for continuous gravitational waves sources are typically computationally limited. eg. Coherent S2 Sco X-1 search limited to observation time of $\sim$ 6 hours.
- New search strategies are required (proposed method adapted from [RCE03]).

LMXB's

- LMXB's consist of a neutron star (NS) (or black hole) in orbit with a lower mass companion star (either main sequence, white dwarf or evolved star).
- The lower mass companion has filled its Roche Lobe and material is being transferred into an accretion disk around the NS. accretion disk.
- These are *not* seen as pulsars so the frquency is unknown (although some exhibit type 1 X-Ray bursts).

GW emission mechanism

Asymmetries in the NS crust caused and sustained indirectly by infalling material [Wag84, UCB00].

Millisecond accreting X-Ray pulsars

- An accreting binary system where pulses in the X-Ray emission are observed at the NS rotation frequency.
- The pulses are generated by infalling material being channelled onto "hotspots" on the NS surface.
- Small subset (7 currently known) have spin periods of order 10^{-3} sec and are known as millisecond (or recycled) pulsars.

GW emission mechanism

Asymmetries in the NS crust caused and sustained indirectly by infalling material [Wag84, UCB00, MP05].

Binary Radio Pulsars

- Radio pulsars in binary systems typically have very well defined orbital and phase parameters but not all of them.
- The work by [PW] and consequent results [Col] leave ~ 40 radio pulsars as unsuitable for the single filter time domain analysis.

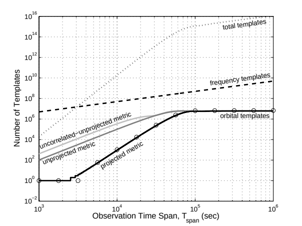
GW emission mechanism

Long term asymmetries in the NS crust [Bil98, UCB00, Cut02]

The fully coherent approach

- Using matched filtering, we perform a search over a bank of templates.
- A metric approach is used to optimally place templates.
- The $\mathcal{F}$-statistic is then computed for each template [JKS98].
- This approach was used for the S2 analysis [Col06].

Accreting binary pulsar



Key Problem for LMXB's

This approach is computationally prohibitive $T_{\mathrm{comp}} \propto T^7$ (for Sco X-1 $T < 10^5$ sec).

The Stack-Slide approach

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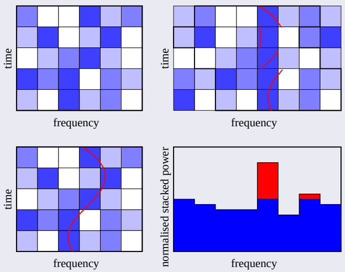
- This is an incoherent search.
- The data are split into M contiguous chunks.
- A coherent search is performed on each chunk.
- The search products are summed (stacked) as a function of source frequency (slid) [BCCS98].

Stack-Slide example



The Radiometer approach

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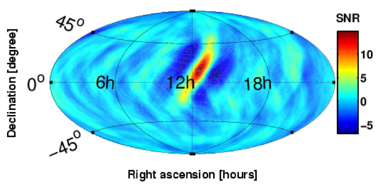
- The radiometer approach has been developed initially for the analysis of the GW stochastic background [Bal06, Col04].
- It can be used to target a particular sky location.
- It cross corrolates data between 2 detectors and uses the signals (if present) as filters.

Radiometer example



A toy model example

Let us define a toy model binary signal as

$$h(t) = g(t)h_0 \cos\left\{2\pi f_0 \left[t + a\sin\left(\frac{2\pi}{P}(t - t_0)\right) + \phi_0\right]\right\},$$

where $g(t)$ is the time domain window function. This can be decomposed in the frequency domain to give

$$\tilde{h}(f) = \tilde{g}(f) * \frac{h_0}{2}\sum_{n=-m}^{m} J_n(2\pi f_0 a)e^{-i(nt_0+\phi_0)}\delta(f - f_n)$$

where $M = 2m + 1 \approx 4\pi f_0 a$. The power is then

$$|\tilde{h}(f)|^2 \approx |\tilde{g}(f)|^2 * \frac{h_0^2}{4}\sum_{n=-m}^{m} J_n^2(2\pi f_0 a)\delta^2(f - f_n)$$

The frequency domain signal

For $T \gtrsim 3P$ the signal power $|\tilde{h}(f)|^2$ is localised in $M \approx 4\pi f_0 a$ "spikes".

- Each "spike" is seperated by $1/P$ Hz
- The relative amplitude of each "spike" is defined by the f_0 and a.
- The power/$\mathcal{F}$-statistic is *independent* of the orbital phase parameter (usually the time of periapse passage t_p).

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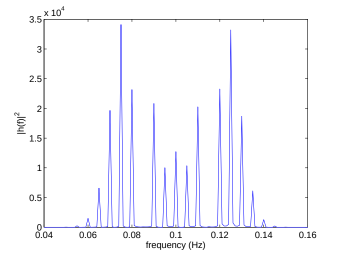
Example signal profile



Dealing with gaps

In the (very likely) situation where the data contains large and frequent gaps in time we deal with this by computing the Fourier transform of the window function $g(t)$.

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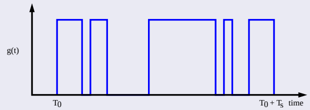
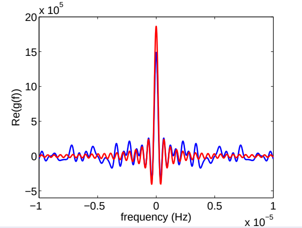
Example $g(t)$



Example $\tilde{g}(f)$



Orbital eccentricity

We can use the Fourier series expansion of the sin and cos of the eccentric anomoly $E(t)$ $(k \in -\infty, \ldots, -1, 1, \ldots \infty)$ eg.

$$\cos E = -\frac{e}{2} + \sum \frac{1}{k} J_{k-1}(ke) \cos k\mathcal{M}.$$

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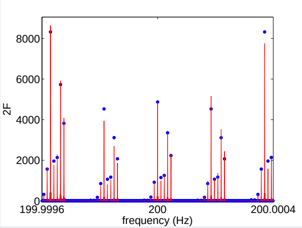
- Non-zero eccentricity acts to spread power amonst existing sidebands.
- Results in change in relative amplitude of spikes *not* location.

Eccentric signal example



Searching with a “comb”

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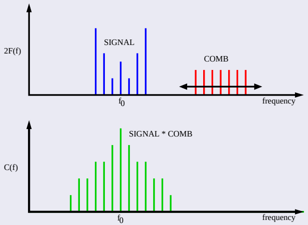
- First compute $\mathcal{F}$-statistic demodulating for sky position only.
- Use a finite sized flat comb $c(f)$ of unit amplitude teeth each spaced by $1/P$ Hz as a template $\rightarrow$ SNR loss $< 35\%$
- The comb will have $M = 4\pi f_0 a$ teeth.

The comb



The detection statistic

$$C(f) = c(f) * 2\mathcal{F}(f) \quad (1)$$

The statistics

The mean and variance of the detection statistic $C(f)$ are given by

$$\langle C \rangle = d_{\rm opt}^2 + 4M, \quad \sigma_C^2 = 4d_{\rm opt}^2 + 4M$$

where $M = 4\pi f_0 a$ and is fixed for a given source and $d_{\rm opt}^2 = h_0^2 T/2S_h$. The SNR of the detection statistic $C(f)$ is therefore

$$\langle \rho_C \rangle \approx \frac{\langle C \rangle - \langle C(h_0 = 0) \rangle}{\sqrt{\sigma_C^2}} \approx \frac{h_0^2 T/2S_h}{2\sqrt{h_0^2 T/2S_h + M}}$$

The 1σ sensitivity is therefore approximately

$$h_0^{(1\sigma)} \approx \begin{cases} 2M^{1/4}\sqrt{S_h/T} & d_{\rm opt}^2 \ll M, \\ 2\sqrt{2S_h/T} & d_{\rm opt}^2 \gg M. \end{cases}$$

The $\mathcal{F}$-statistic likelihood

The $\mathcal{F}$-statistic components F_a and F_b are calculated using

$$F_a = \int x(t)a(t)e^{-i\Phi(t)}dt \qquad F_b = \int x(t)b(t)e^{-i\Phi(t)}dt.$$

Let $F_a = \mathbb{F}_1 + i\mathbb{F}_3$ and $F_b = \mathbb{F}_2 + i\mathbb{F}_4$

$$\langle\mathbb{F}_1\rangle = T\left(AA_1 + CA_2\right)/4, \qquad \langle\mathbb{F}_2\rangle = T\left(CA_1 + BA_2\right)/4,$$
$$\langle\mathbb{F}_3\rangle = -T\left(AA_3 + CA_4\right)/4, \qquad \langle\mathbb{F}_4\rangle = -T\left(CA_3 + BA_4\right)/4,$$

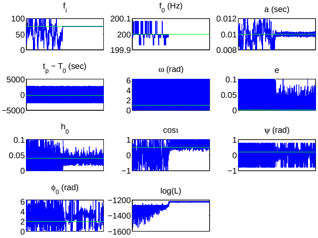
and the covariance matrix governing $\mathbb{F}$ is

$$\sigma^2(\mathbb{F}) = \begin{pmatrix} \mathcal{C} & \mathcal{O} \\ \mathcal{O} & \mathcal{C} \end{pmatrix} \qquad \mathcal{C} = \frac{1}{8}T\langle S_h\rangle \begin{pmatrix} A & C \\ C & B \end{pmatrix},$$

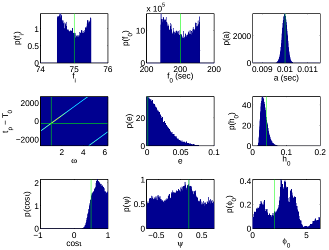
MCMC parameter estimation

An MCMC is performed over the frequency, orbital and nuisance parameters using only $\mathbb{F}$ data located at the sideband frequencies.

MCMC chains

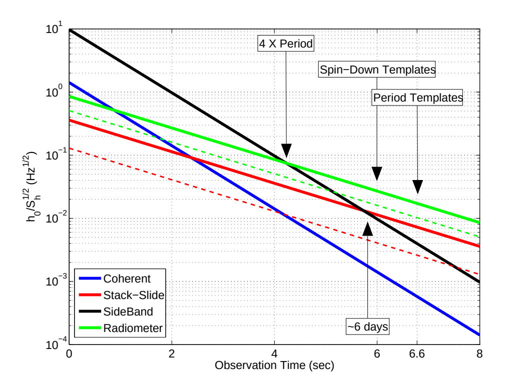


MCMC posteriors



Comparison of methods

Search method comparison (VERY ROUGH)



How fast is this search?

- The following scaling is true when not searching over sky position, period or spin down (3GHz Desktop):

$$\mathcal{F}\text{-statistic}: \; T_{\mathcal{F}} \sim 500 \text{ sec} \left(\frac{T}{10^6\text{sec}}\right)^2 \left(\frac{f_{\text{band}}}{1\text{Hz}}\right)$$

$$C(f): \; T_C \sim 2 \text{ sec} \left(\frac{T}{10^6\text{Sec}}\right) \left(\frac{f_{\text{band}}}{1\text{Hz}}\right) \ln\left[\left(\frac{T}{10^6\text{Sec}}\right) \left(\frac{f_{\text{band}}}{1\text{Hz}}\right)\right].$$

- Stack-Slide run time for a single template is similar.
- For stack-slide to compete it needs to search over many orbital templates. ie. slowing it down by the number of templates.
- At present the majority of the search time is spent in the MCMC stage and is strongly dependent upon the threshold set on $C(f)$.

Conclusions and future work

- Pipeline improvements
  - Extend the analysis to make use of the multi-IFO $\mathcal{F}$-statistic.
  - Incorporate spin up/down into the MCMC.
  - Look at applicability to LISA as an all sky search for white dwarf binaries.
- Analysis plans
  - We plan to analyse all LMXB's with known period and sky position.
  - To do a simple single frequency MCMC search for the accreting millisecond X-Ray pulsars ($T < 10^6$ sec) (coherent).
  - Assess which of the surplus radio pulsars are suitable and perform the MCMC search on these.

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