Preliminary Results of LIGO-ALLEGRO Stochastic Background Search

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on behalf of the LIGO Scientific Collaboration
and the ALLEGRO Group

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Outline

I Background/Motivation for LLO-ALLEGRO Search

- LLO-ALLEGRO Pair (proximity, overlap modulation)
- Technical Considerations (sampling, heterodyning)

II S4 Data Analysis

- Data Volume by Orientation
- Validation: Software & Hardware Injections
- Preliminary Cross-Correlation Results
- Statistical Interpretation: Upper Limit

Sensitivity to Stochastic GW Backgrounds

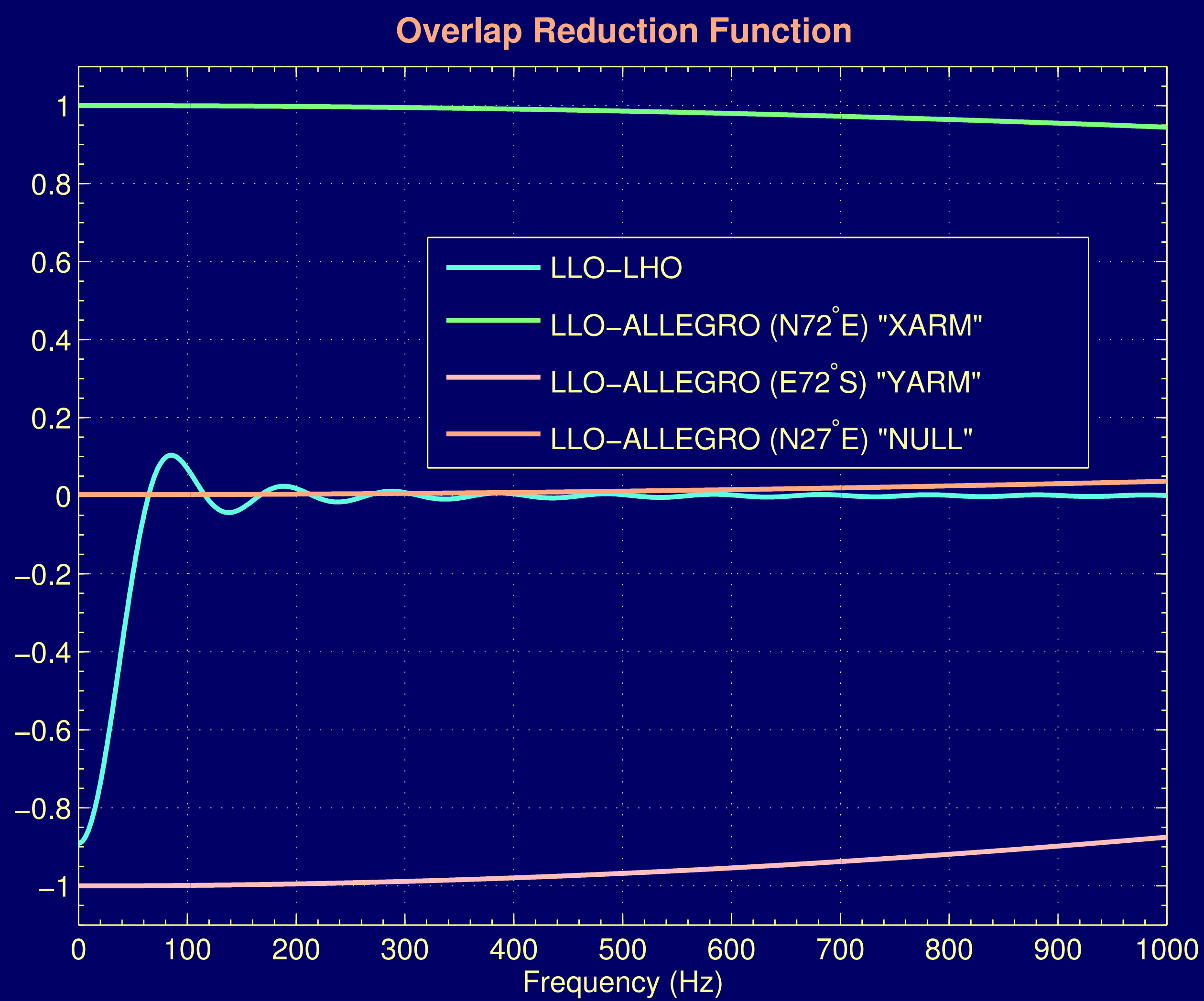
- Optimally filtered CC statistic

$$Y = \int df \, \underbrace{\widetilde{s}_1^*(f) \, \widetilde{Q}(f) \, \widetilde{s}_2(f)}_{Y(f)}$$

- Optimal filter $\widetilde{Q}(f) \propto \frac{S_{\text{gw}}(f)\gamma_{12}(f)}{P_1(f)P_2(f)}$
  (Initial analyses assume
  $S_{\text{gw}}(f)$ or $\Omega_{\text{gw}}(f) \propto f^3 S_{\text{gw}}(f)$ constant across band)
- Optimally filtered cross-correlation method has Ω_{gw} sensitivity

$$\sigma_\Omega \propto \left(T \int \frac{df}{f^6} \frac{\gamma_{12}^2(f)}{P_1(f)P_2(f)} \right)^{-1/2}$$

- Significant contributions when
  - detector noise power spectra $P_1(f)$, $P_2(f)$ small
  - overlap reduction function $\gamma_{12}(f)$ (geom correction) near ± 1



LLO-ALLEGRO only $\sim 40\,\mathrm{km}$ apart $\longrightarrow$ still sensitive @ $900\,\mathrm{Hz}$

Response different for XARM, YARM, NULL orientations

ALLEGRO ran in all 3 orientations during LIGO S4 Run (2005 Feb 22-Mar 23)

LLO-ALLEGRO: Technical Considerations

- LIGO data digitally downsampled 16384 Hz → 4096 Hz
  ALLEGRO data heterodyned at 904 Hz & sampled at 250 Hz

- Heterodyning means CC stat complex:

$$Y = \int_{f_{\text{min}}}^{f_{\text{max}}} df\, \widetilde{s}_1^*(f)\, \widetilde{Q}(f)\, \widetilde{s}_2(f)$$

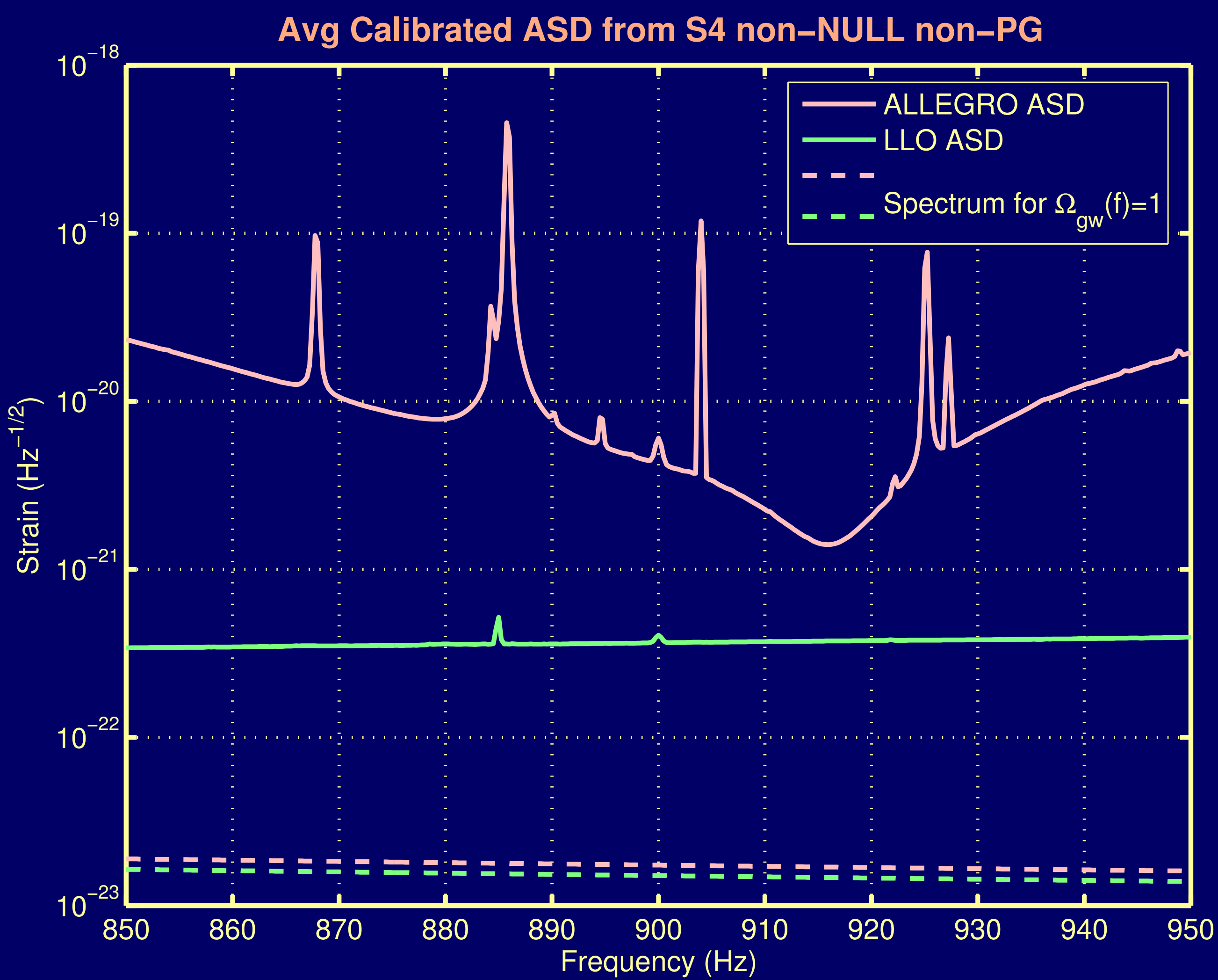
  real part Gaussian-distributed about SGWB strength;
  imag part Gaussian-distributed about 0.

- Differently-sampled data correlated in freq domain
  → Method written up in *CQG* **22**, S1087 (2005)

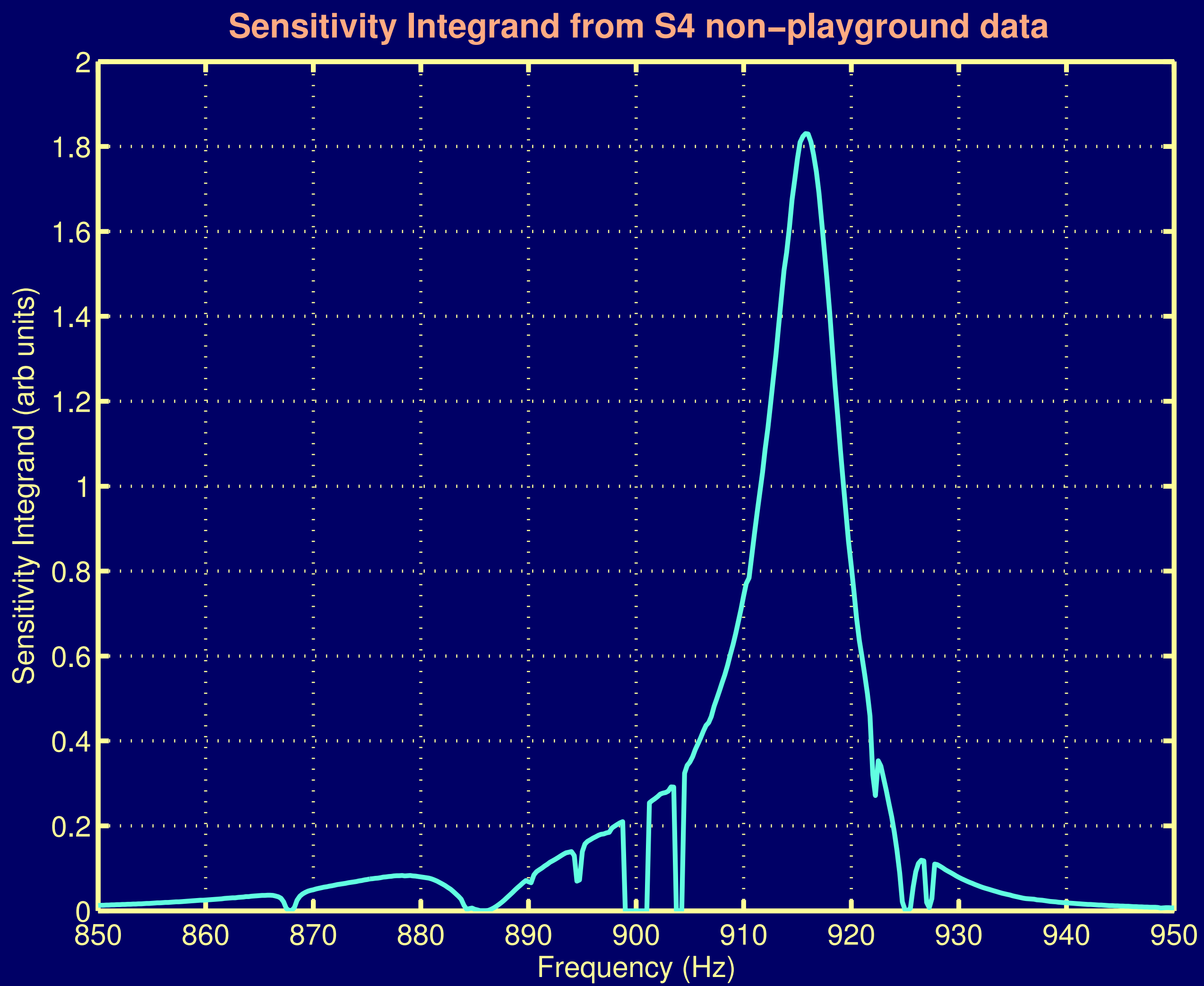
LLO-ALLEGRO data from LIGO S4 Run

- $\sim$ 10% of data set aside as "playground";
  coïnc Non-PG data surviving DQ vetoes divided into 60s segs;
  Incoherent stationarity cut applied to reject segs
  where sensitivity changing too rapidly
  (need stationarity for well-behaved optimal filter)

- Non-playground data in 3 orientations:
  - "NULL" ($0.023 < \gamma(f) < 0.029$): 88.2 hr after cuts
    "off-source" data useful for data quality & cross-checks
  - "YARM" ($-0.89 > \gamma(f) > -0.91$): 114.7 hr after cuts
  - "XARM" ($0.95 < \gamma(f) < 0.96$): 181.2 hr after cuts



Frequency band determined by ALLEGRO noise curve

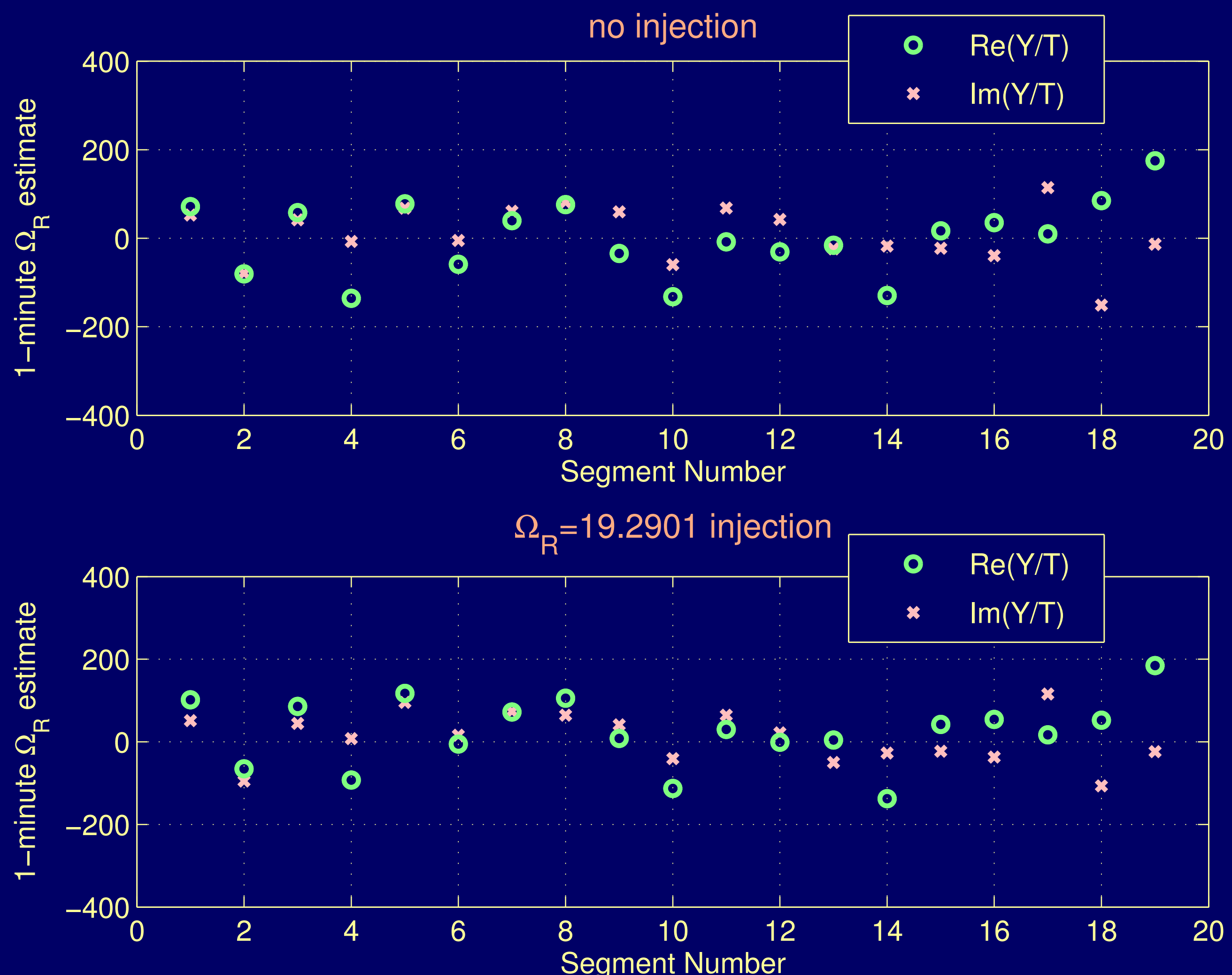


Most of sensitivity from 905–925 Hz

Software Injections into S4 Playground

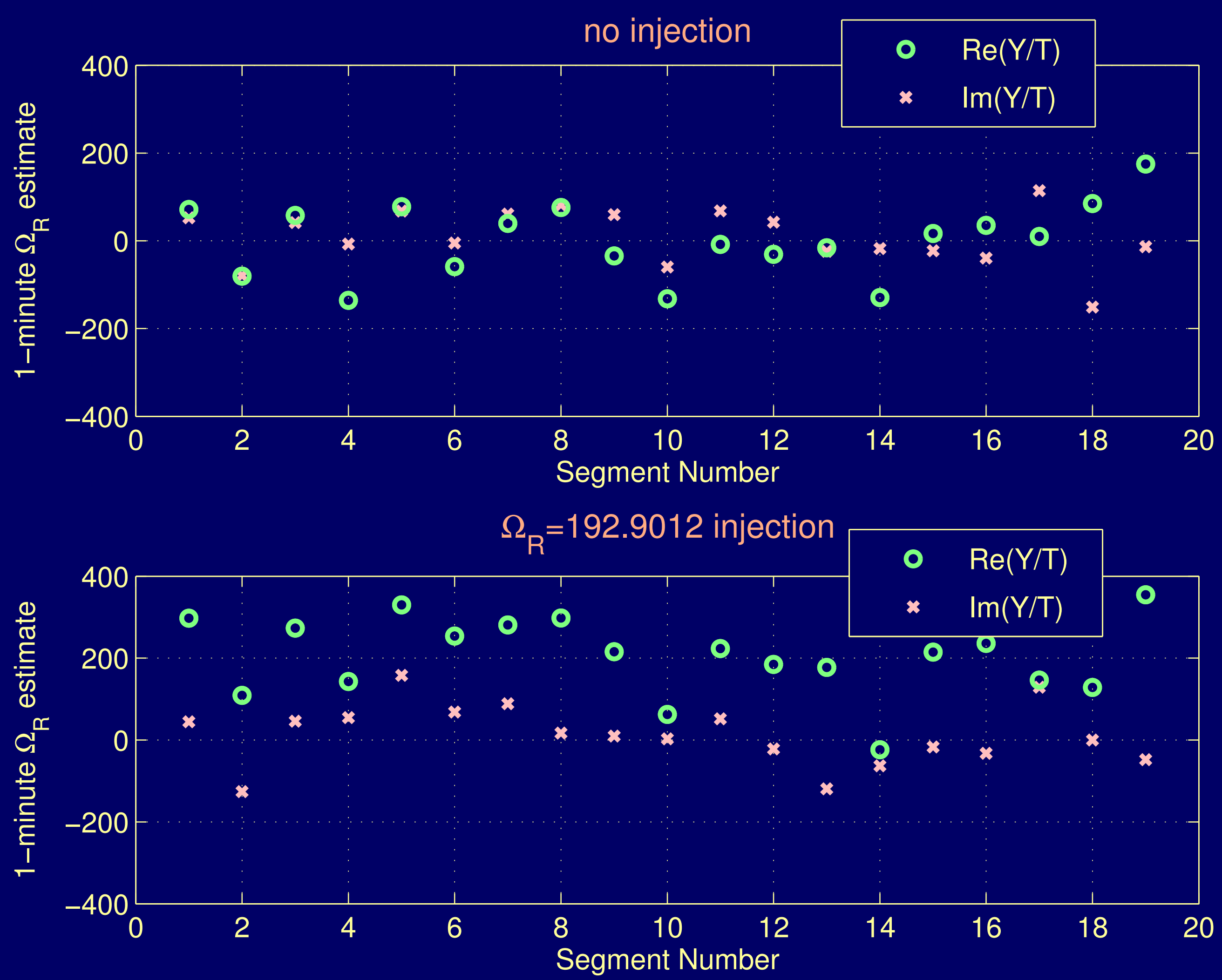
- Combined 90% error bars for all playground data ~ 2
- Inject simulated signals of strength $\Omega_R = 1.9,\ 3.9,\ 9.6,\ 19.$
- Note: individual jobs have error bars around 120.
  SW injections only detectable over time.

Stats w/ & w/o SW Inj (19 60-sec segs)

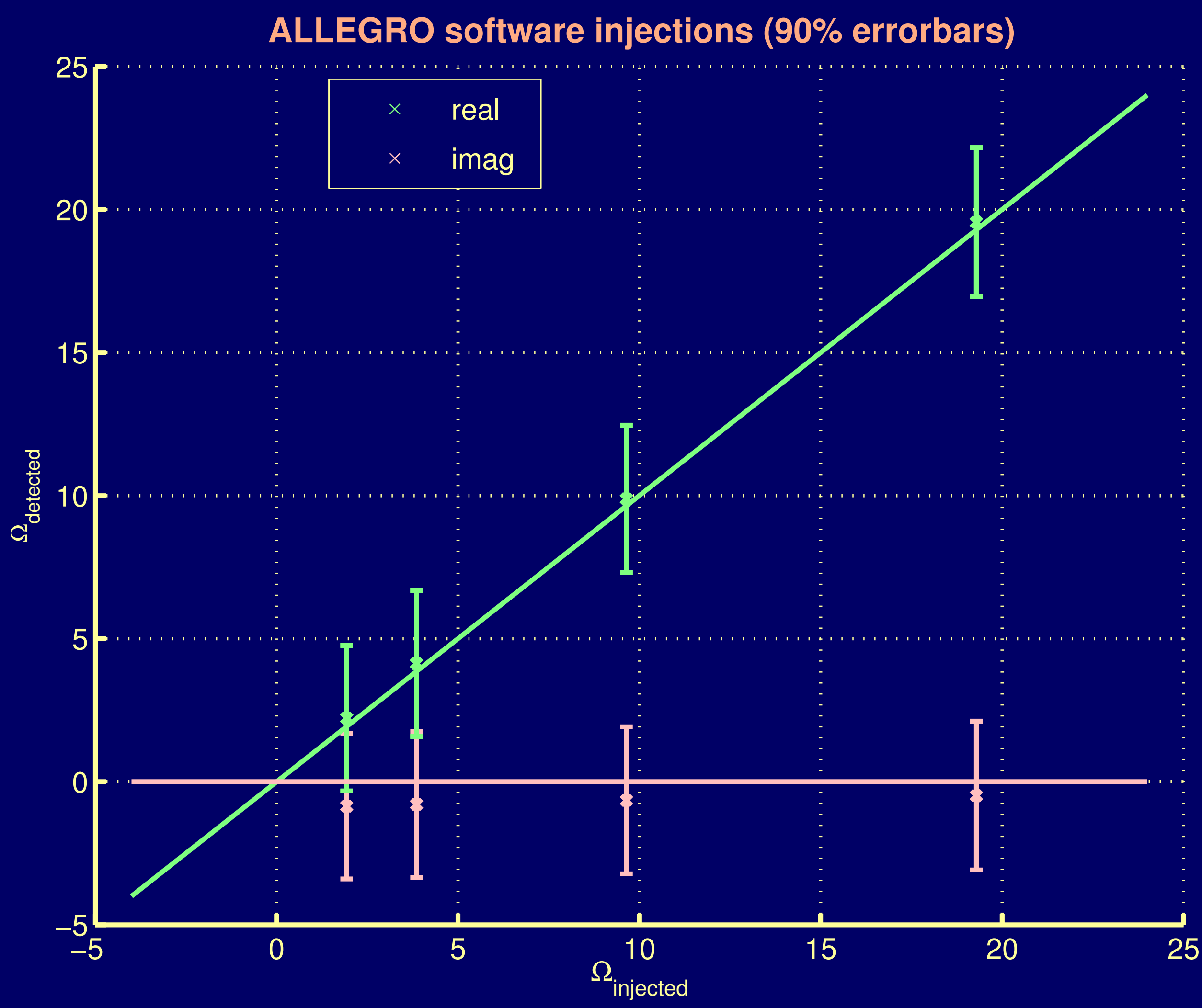


Injecting $\Omega(f) = 19.3$ has negligible impact on minute-by-minute correlations

Stats w/ & w/o SW Inj (19 60-sec segs)



Compare $\Omega(f) = 193$ injection, which is visible minute-by-minute



$\Omega(f) = 3.9$, 9.6, 19 injections recovered from full PG

($\Omega(f) = 1.9$ just at threshold of detectability)

Note: injected same random signals w/different amplitudes into same noise

S4 Hardware Injections

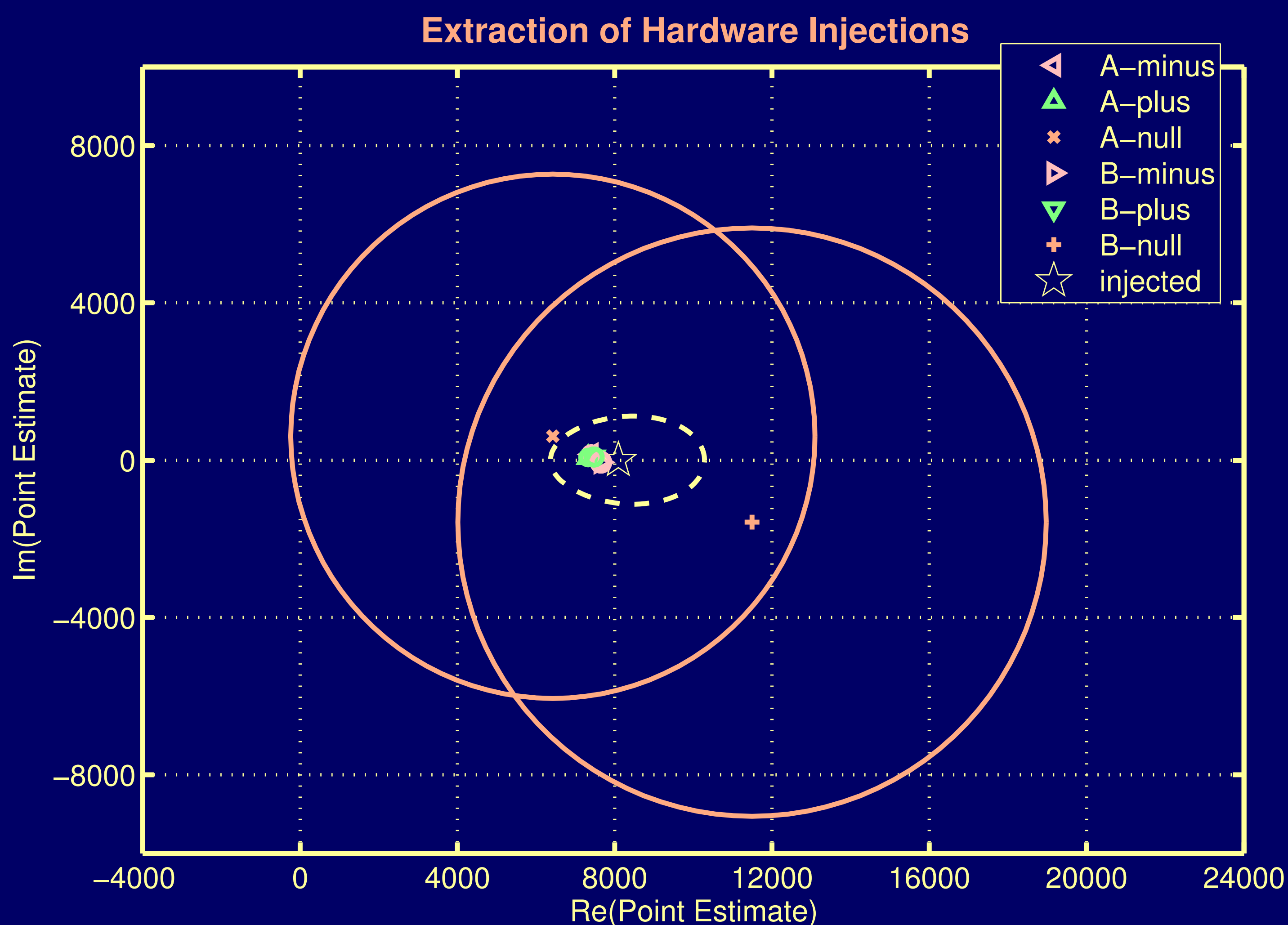
- 1024-second simulated signals injected into LLO & ALLEGRO hardware a total of nine times. Simulated all three orientations.

- One "round" of three injections had non-const $\Omega_{\mathrm{gw}}(f)$

- Other two rounds ("A" & "B") injected const $\Omega_{\mathrm{gw}}(f) = 8100$ $\longrightarrow$ Focus on those

- Sensitivity of cross-correlation to injections simulating XARM ("plus") and YARM ("minus") is comparable

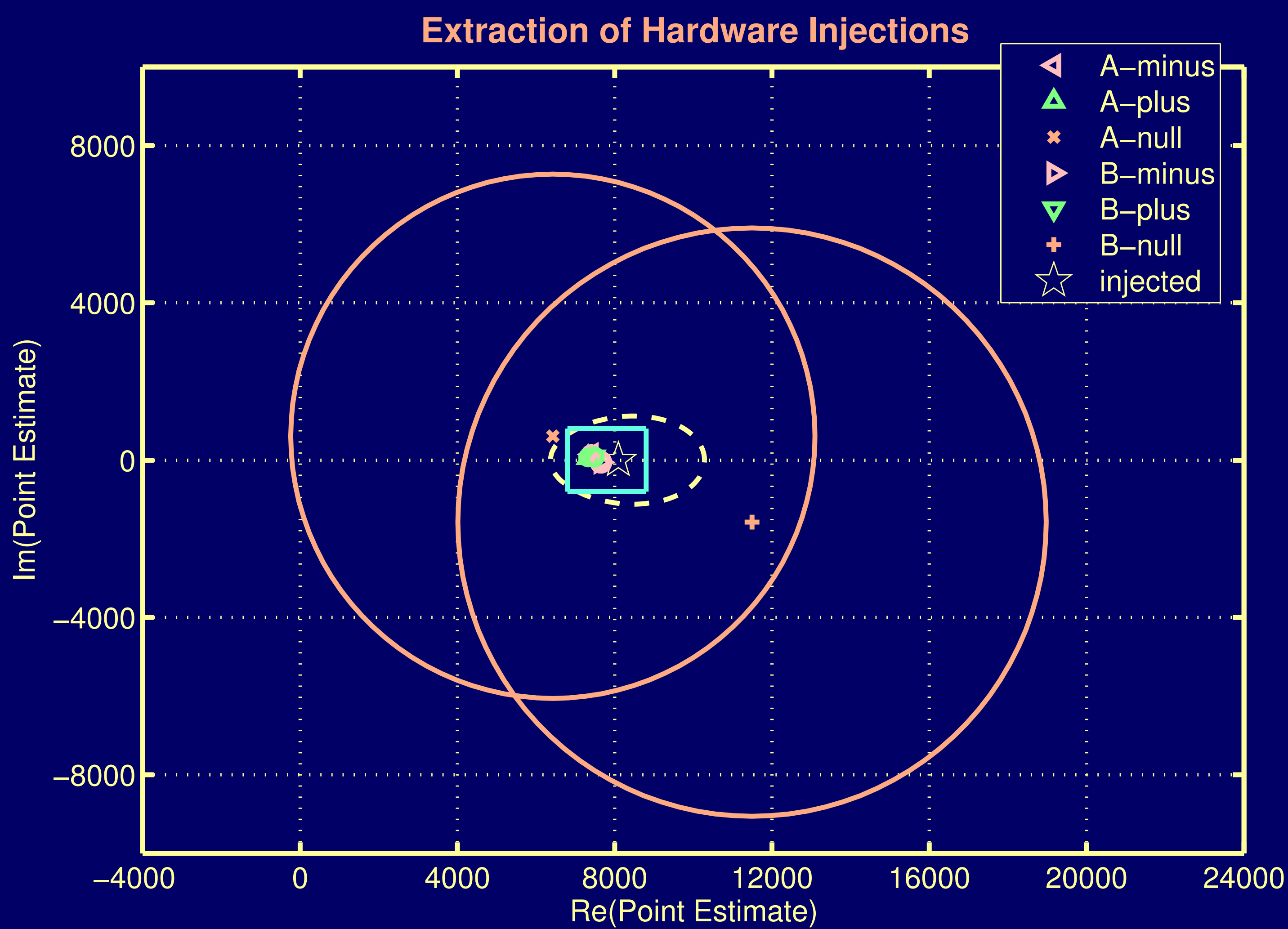
- "null" injection less correlated b/c of simulated misalignment



Circles: 90% statistical uncertainty (null measurements less sensitive)

90% dashed calib uncertainty "teardrop" around $\Omega_R = 8100$

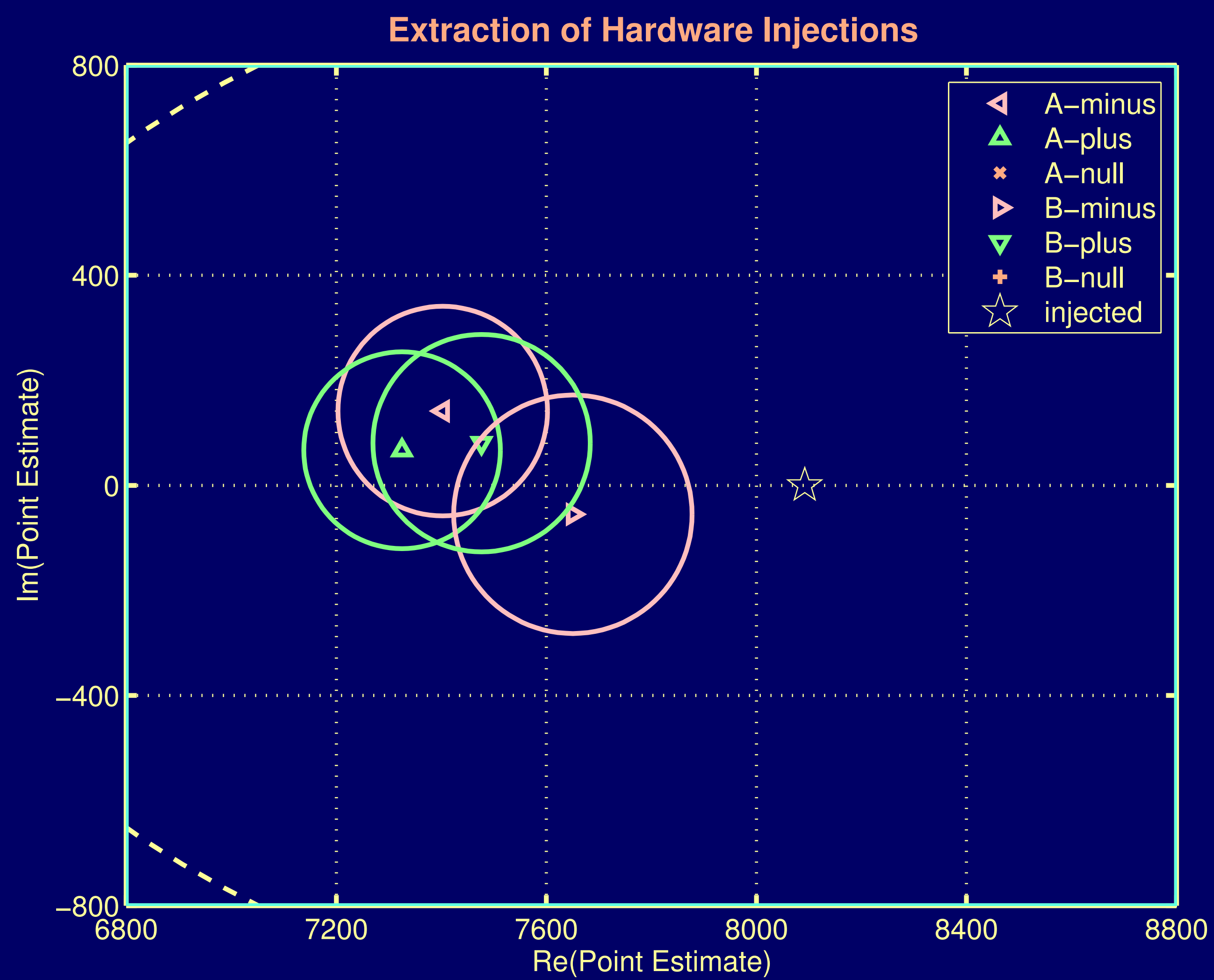
HW injections recovered consistent w/cal uncertainty



Circles: 90% statistical uncertainty (null measurements less sensitive)

90% dashed calib uncertainty "teardrop" around $\Omega_R = 8100$

HW injections recovered consistent w/cal uncertainty   Zoom in on blue box ...



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90% dashed calib uncertainty "teardrop" around $\Omega_R = 8100$

Systematic offset $<$ cal uncertainty

S4 Preliminary Cross-Correlation Results

Optimally filter looking for const $\Omega_{\text{gw}}(f) \equiv \Omega_R$
Assume $H_0 = 72\,\text{km/s/Mpc}$ (so $\Omega_R = h_{72}^2 \Omega_{\text{gw}}(f)$)

Analyzed non-playground data w/overlapping 60-sec Hann windows:

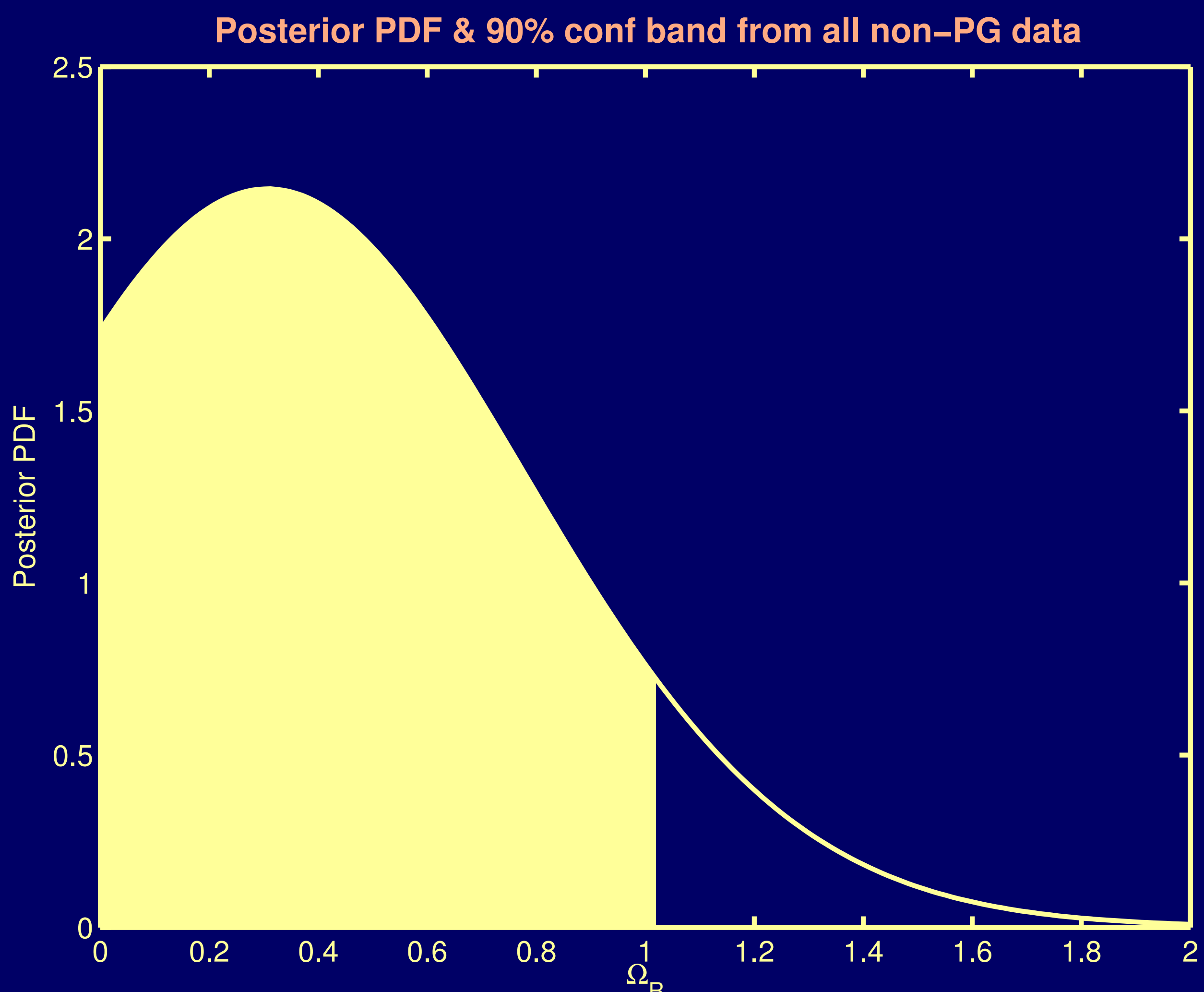
Type	T_{eff} (hrs)	Ω_R Point Estimate	Ω_R Error Bar
XARM	181.2	$0.61 + 0.25i$	0.56
YARM	114.7	$-0.47 + 0.47i$	0.90
non-NULL	295.8	$0.31 + 0.31i$	0.48
NULL	88.2	$10.96 - 43.89i$	28.62
all	384.1	$0.31 + 0.30i$	0.48

No correlation observed
$\longrightarrow$ Convert CC meas of $0.31 + 0.30i$ & theor errorbar of 0.48 into upper limit . . .

Constructing Bayesian Posterior PDF

- Formal prior on $\Omega_{\text{gw}}(915\,\text{Hz})$
  from Explorer-Nautilus: uniform on $[0, 115]$

- Marginalize likelihood fcn over calibration uncertainty:
  L1 5% amp, 2° phase; A1 10% amp, 3° phase.
  (Assume Gaussian prior in ln(amp) and phase.)



prelim 90% CL UL: $\Omega_R < 1.02$ i.e., $\sqrt{S_{\text{gw}}(915\,\text{Hz})} < 1.5 \times 10^{-23}\,\text{Hz}^{-1/2}$

100× improvement on $\Omega_{\text{gw}}(907\,\text{Hz}) < 115$ [$h_{100}^2\Omega_{\text{gw}}(907\,\text{Hz}) < 60$]

from NAUTILUS-EXPLORER [Astone et al., A & A **351**, 811 (1999)]

LLO-ALLEGRO: Summary

- First stochastic measurement correlating bar w/ifo data;
  Probes higher frequency band than LLO-LHO: $\sim 850 - 950\,\text{Hz}$

- Diff orientations of ALLEGRO $\longrightarrow$ different stochastic response
  (Data taken in 3 orientations during S4)

- Preliminary S4 upper limit results from $\sim 370\,\text{hrs}$ of data:
  $\sqrt{S_{\text{gw}}(915\,\text{Hz})} < 1.5 \times 10^{-23}\,\text{Hz}^{-1/2}$
  I.e., $\Omega_{\text{gw}}(915\,\text{Hz}) < 1.02$ $[h_{100}^2\Omega_{\text{gw}}(915\,\text{Hz}) < 0.53]$,
  $100\times$ better than EXPLORER-NAUTILUS (previous high freq UL)

- Analysis extracts long-time, low-amplitude simulated signals
  (software injections)

- Hardware inj extracted consistent w/calibration uncertainty

Extra Slides

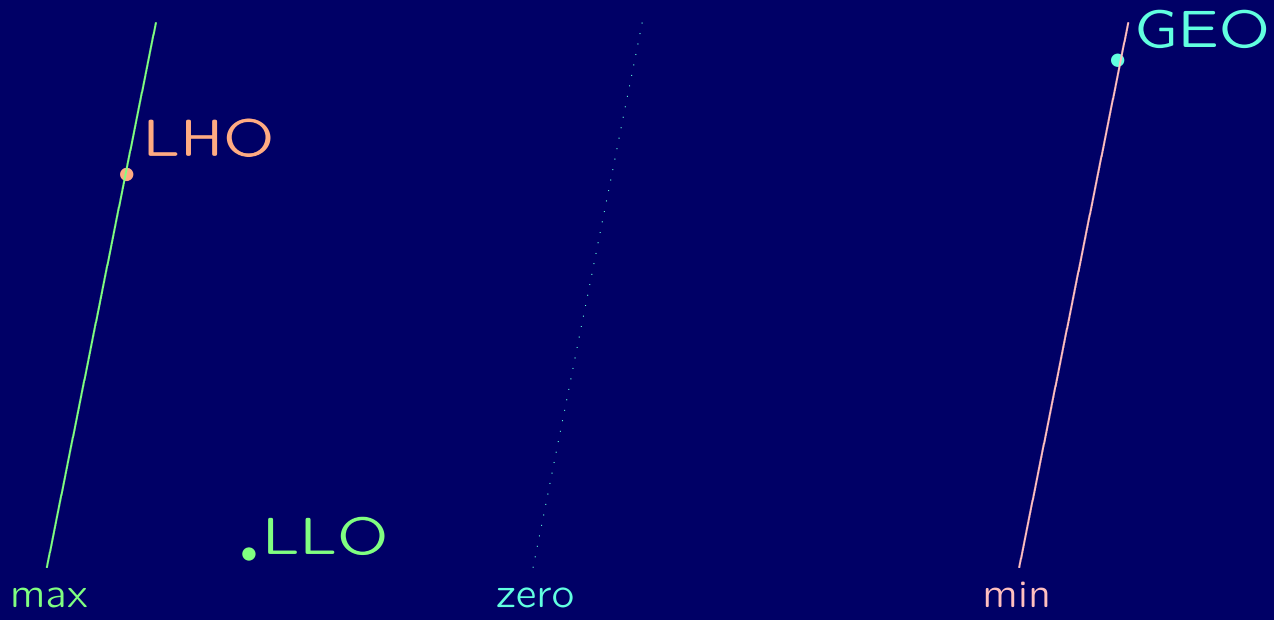
Overlap Reduction Function

$$\gamma_{12}(f) = d_{1ab} d_2^{cd} \frac{5}{4\pi} \iint_{S^2} d^2\Omega_{\hat{\mathbf{n}}}\ P^{\mathrm{TT}}{}^{ab}_{cd}(\hat{\mathbf{n}}) e^{i2\pi f \hat{\mathbf{n}}\cdot\Delta\vec{\mathbf{r}}/c}$$

Depends on alignment of detectors (polarization sensitivity)

Frequency dependence from cancellations when $\lambda \lesssim$ distance

$\longrightarrow$ Widely separated detectors less sensitive at high frequencies



This wave drives LHO & GEO out of phase

Overlap Reduction Function

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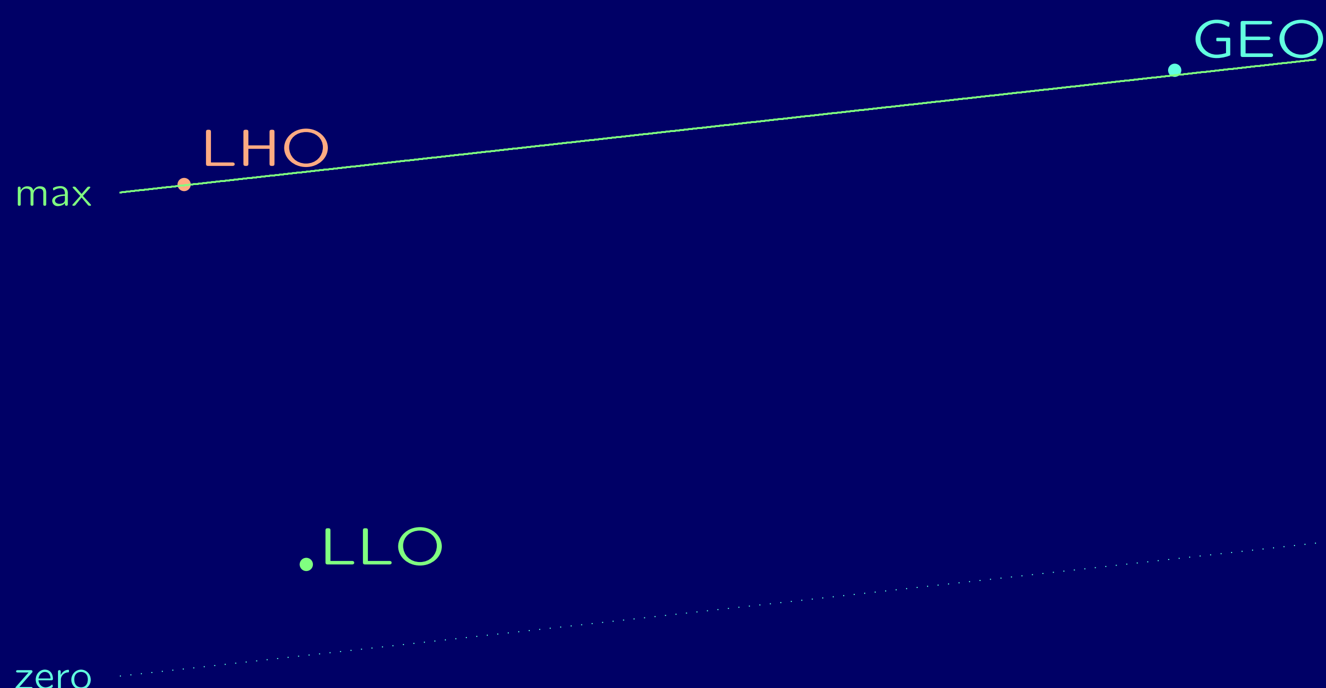


This wave (same λ) drives LHO & GEO in phase

Constructing Posterior PDF

- Overall estimate $\widehat{\Omega}_R = x + iy$ has likelihood function (for given actual $\Omega_R = \Omega_{\text{gw}}(915\,\text{Hz})$)

$$P(x, y|\Omega_R, \sigma_\Omega) \propto \exp\left(-\frac{|x + iy - \Omega_R|^2}{2{\sigma_\Omega}^2}\right)$$

- Bayes's theorem gives posterior PDF

$$P(\Omega_R|x, y, \sigma_\Omega) = \frac{P(x, y|\Omega_R, \sigma_\Omega)P(\Omega_R)}{P(x, y|\sigma_\Omega)}$$
$$\propto e^{-(x-\Omega_R)^2/2{\sigma_\Omega}^2} P(\Omega_R)$$

Note imag part y of pt est factors out

Marginalization Over Calibration Uncertainty

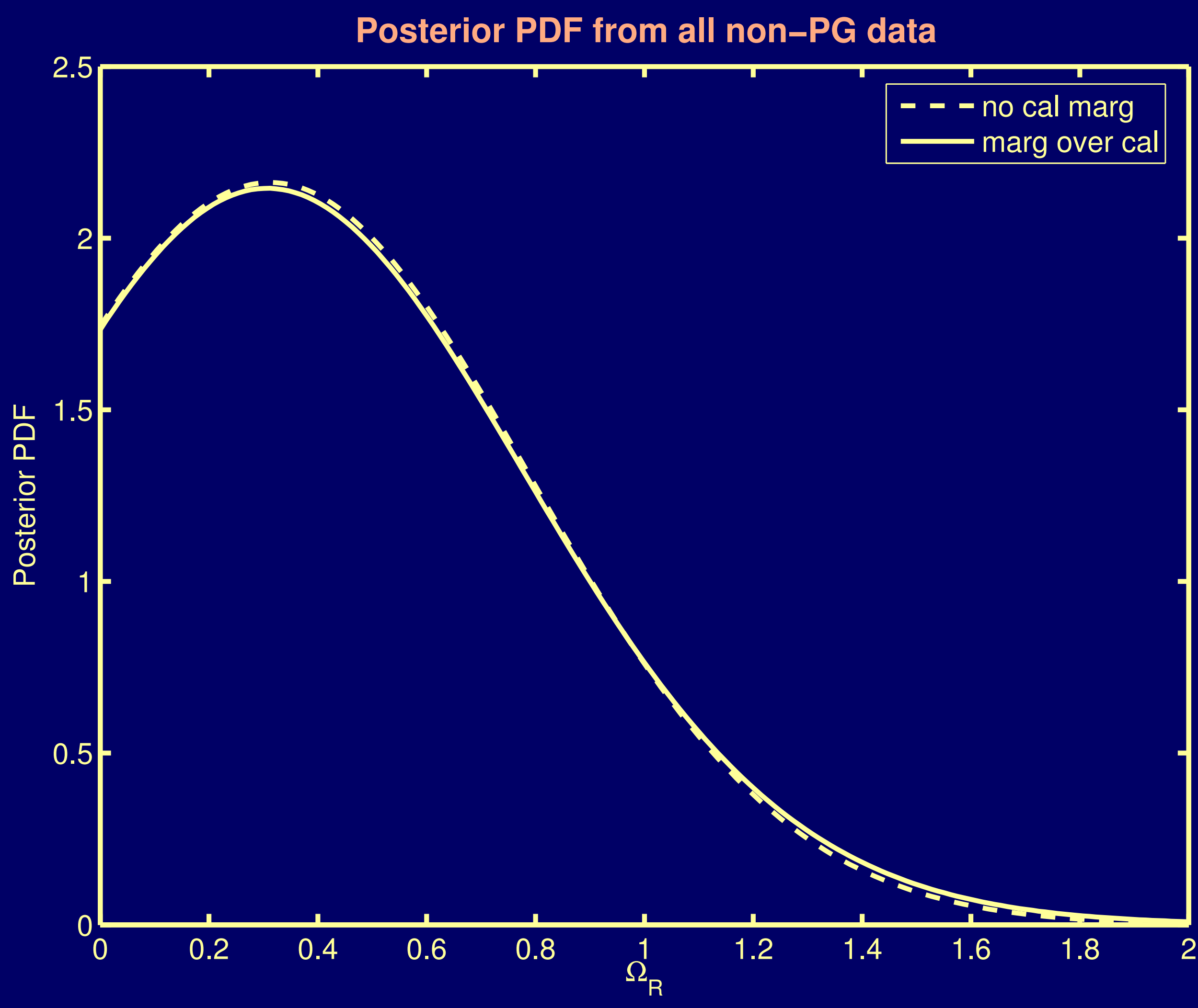
- Calibration of LLO & ALLEGRO uncertain in amp & phase
  Marginalize over unknown correction factor $e^{\Lambda+i\phi}$:

$$P(x,y|\Omega_R,\sigma_\Omega,\Lambda,\phi) \propto \exp\left(-\frac{\left|x+iy-\Omega_R e^{\Lambda+i\phi}\right|^2}{2{\sigma_\Omega}^2}\right)$$

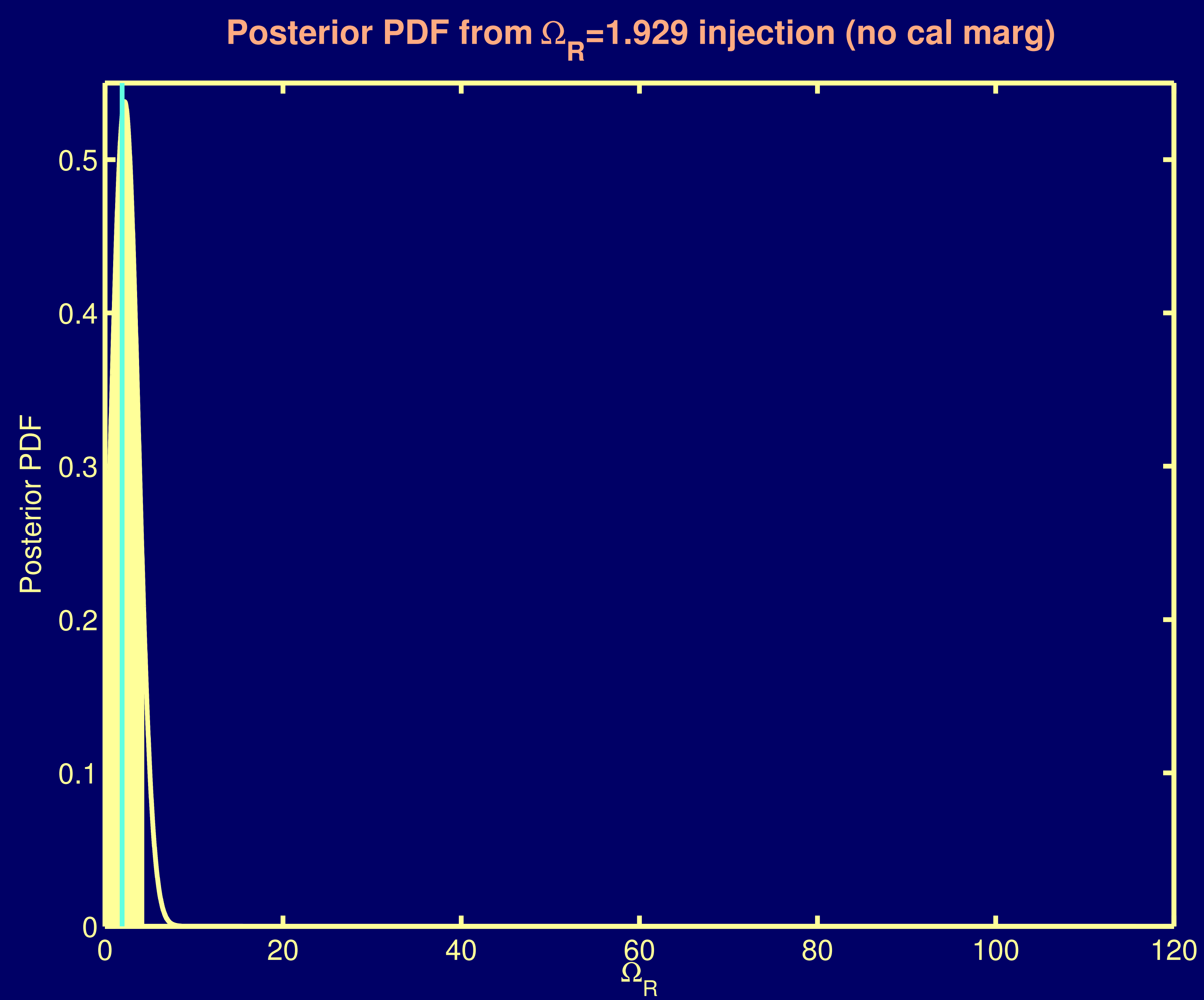
so the posterior after marginalizing the likelihood function is

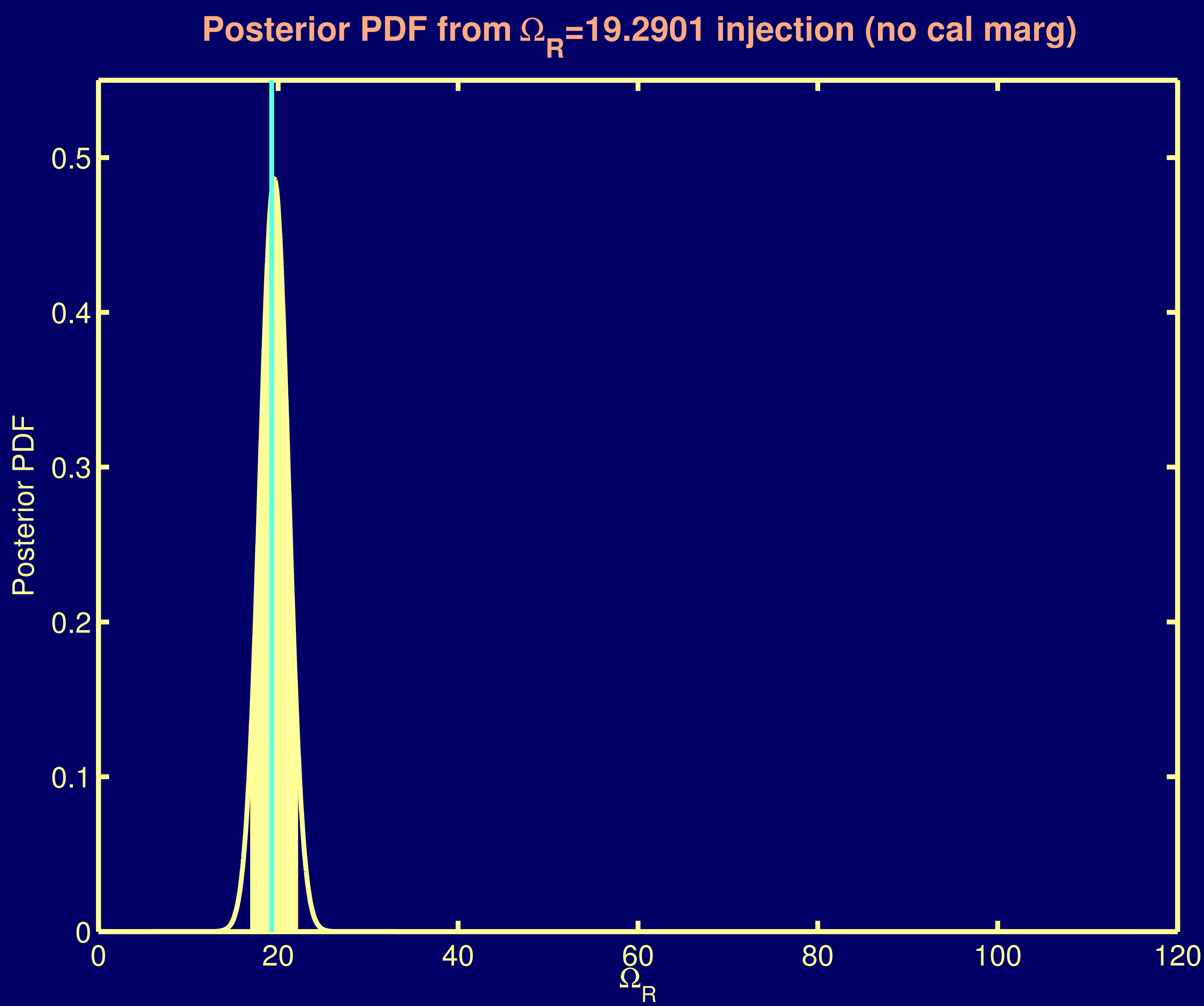
$$P(\Omega_R|x,y,\sigma_\Omega) \propto \int_{-\infty}^{\infty} d\Lambda \int_{-\pi}^{\pi} d\phi\, \exp\left(-\frac{\left|x+iy-\Omega_R e^{\Lambda+i\phi}\right|^2}{2{\sigma_\Omega}^2}\right) P(\Lambda,\phi)\, P(\Omega_R)$$

which does depend on imag part y



Cal marginalization doesn't matter much @ low SNR


Posterior PDF from Ω_R=1.929 injection (no cal marg)
Posterior PDF
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
Ω_R


Posterior PDF from Ω_R=19.2901 injection (no cal marg)
Posterior PDF
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
Ω_R

Time-Shift Analyses

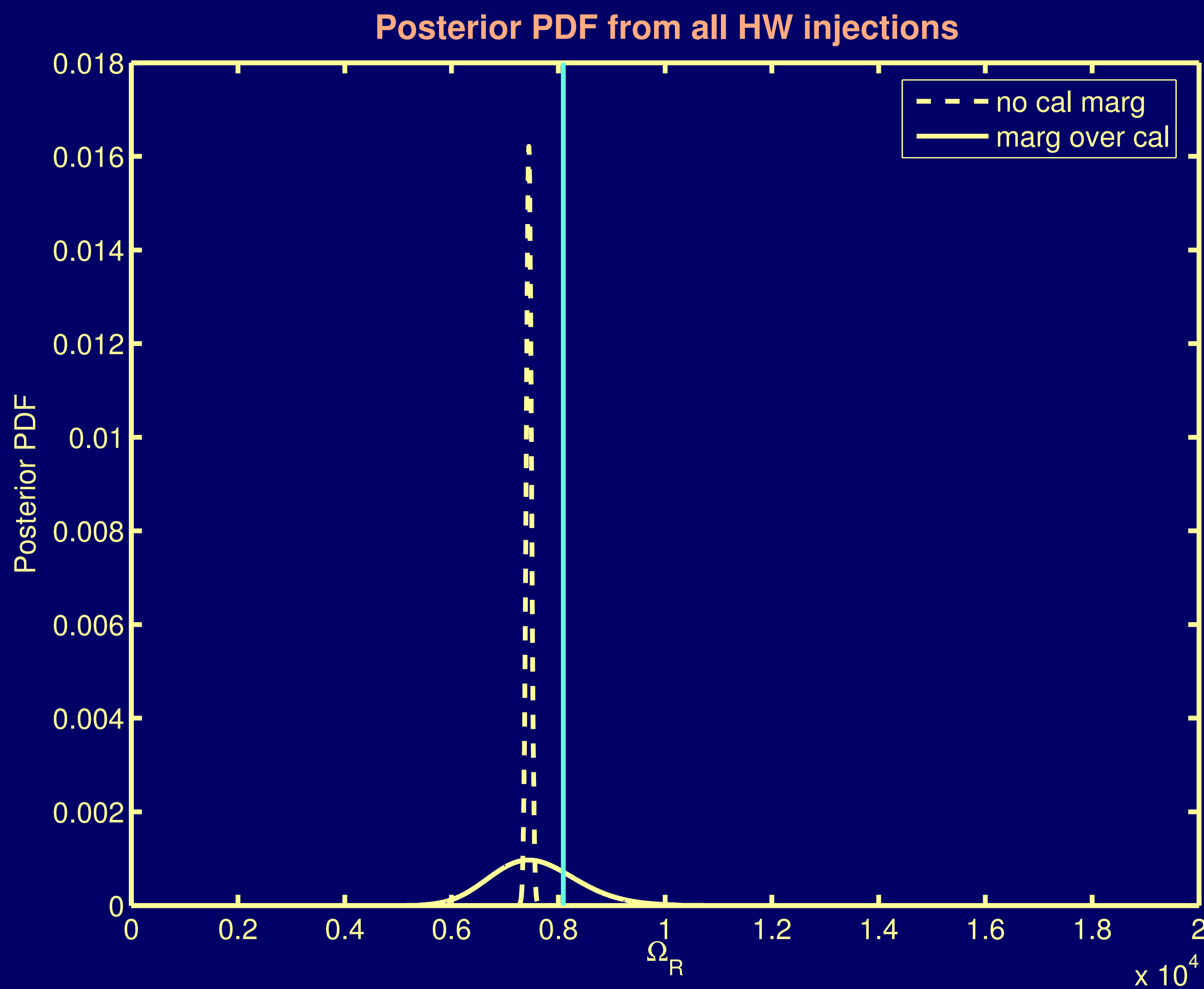
- Learned about timing issues via HW injections:
  Time-shift analysis helped resolve issues w/ALLEGRO timing
  Also revealed sample-and-hold & other digital effects
  in injection system which introduce relative time shift of
  $\frac{1}{2\times 4096\,\text{Hz}} - 18\,\mu\text{s} = 104\,\mu\text{s}$

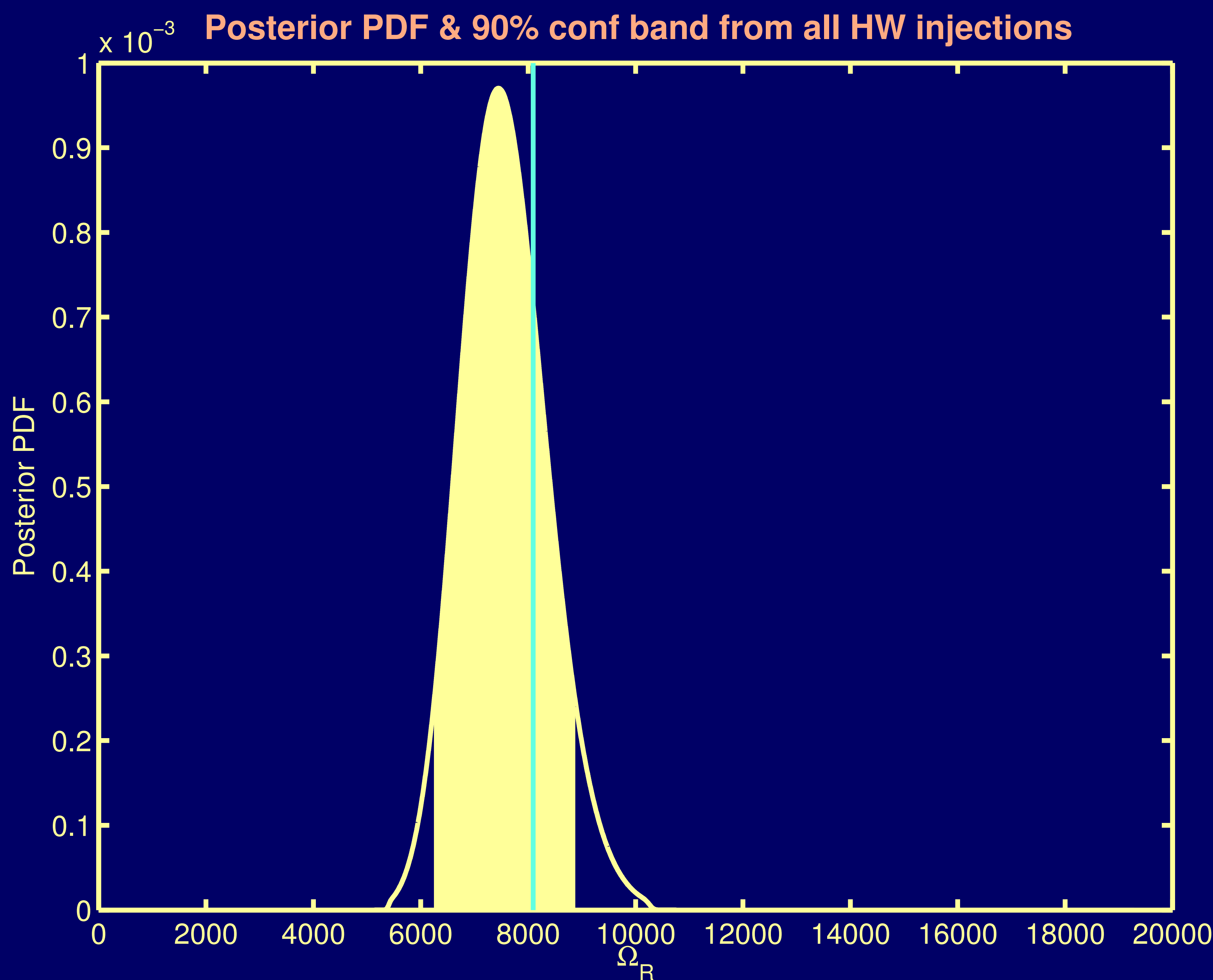
- Post-processing correction:
  Simulate small timeshift w/freq-dependent phase shift

  $$Y(f) \longrightarrow Y(f)\, e^{i2\pi f\tau}$$

  inv FT of CC integrand gives CC values as fcn of time-shift:

  $$Y(\tau) = \int_{f_{\text{min}}}^{f_{\text{max}}} df\, Y(f)\, e^{i2\pi f\tau}$$


Posterior PDF from all HW injections
no cal marg
marg over cal
Posterior PDF
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Ω_R
x 10^4


Posterior PDF & 90% conf band from all HW injections
x 10^-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Posterior PDF
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
Ω_R