Preliminary Results of LIGO-ALLEGRO Stochastic Background Search

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Outline

I Background/Motivation for LLO-ALLEGRO Search

- LLO-ALLEGRO Pair (proximity, overlap modulation)
- Technical Considerations (sampling, heterodyning)

II S4 Data Analysis

- Data Volume by Orientation
- Validation: Software & Hardware Injections
- Preliminary Cross-Correlation Results
- Statistical Interpretation: Upper Limit
Sensitivity to Stochastic GW Backgrounds

- Optimally filtered CC statistic

\[ Y = \int df \frac{\tilde{s}_1^*(f) \tilde{Q}(f) \tilde{s}_2(f)}{Y(f)} \]

- Optimal filter \( \tilde{Q}(f) \propto \frac{S_{gw}(f) \gamma_{12}(f)}{P_1(f)P_2(f)} \)

(Initial analyses assume \( S_{gw}(f) \) or \( \Omega_{gw}(f) \propto f^3 S_{gw}(f) \) constant across band)

- Optimally filtered cross-correlation method has \( \Omega_{gw} \) sensitivity

\[ \sigma_{\Omega} \propto \left( T \int df \frac{\gamma_{12}^2(f)}{f^6 P_1(f)P_2(f)} \right)^{-1/2} \]

- Significant contributions when
  - detector noise power spectra \( P_1(f), P_2(f) \) small
  - overlap reduction function \( \gamma_{12}(f) \) (geom correction) near \( \pm 1 \)
LIGO-ALLEGRO only ~ 40 km apart → still sensitive @ 900 Hz
Response different for XARM, YARM, NULL orientations
ALLEGRO ran in all 3 orientations during LIGO S4 Run (2005 Feb 22-Mar 23)
LLO-ALLEGRO: Technical Considerations

- LIGO data digitally downsampled 16384 Hz → 4096 Hz
  ALLEGRO data heterodyned at 904 Hz & sampled at 250 Hz

- Heterodyning means CC stat complex:
  \[ Y = \int_{f_{\text{min}}}^{f_{\text{max}}} df \tilde{s}_1^*(f) \tilde{Q}(f) \tilde{s}_2(f) \]
  real part Gaussian-distributed about SGWB strength;
  imag part Gaussian-distributed about 0.

- Differently-sampled data correlated in freq domain
  → Method written up in CQG 22, S1087 (2005)
LLO-ALLEGRO data from LIGO S4 Run

- ~ 10% of data set aside as “playground”;
  coinc Non-PG data surviving DQ vetoes divided into 60s segs;
  Incoherent stationarity cut applied to reject segs
  where sensitivity changing too rapidly
  (need stationarity for well-behaved optimal filter)

- Non-playground data in 3 orientations:
  - “NULL” (0.023 < \(\gamma(f)\) < 0.029): 88.2 hr after cuts
    “off-source” data useful for data quality & cross-checks
  - “YARM” (−0.89 > \(\gamma(f)\) > −0.91): 114.7 hr after cuts
  - “XARM” (0.95 < \(\gamma(f)\) < 0.96): 181.2 hr after cuts
Frequency band determined by ALLEGRO noise curve
Most of sensitivity from 905–925 Hz
Software Injections into S4 Playground

- Combined 90% error bars for all playground data $\sim 2$

- Inject simulated signals of strength $\Omega_R = 1.9, 3.9, 9.6, 19$.

- Note: individual jobs have error bars around 120. SW injections only detectable over time.
Stats w/ & w/o SW Inj (19 60-sec segs)

Injecting $\Omega(f) = 19.3$ has negligible impact on minute-by-minute correlations
Stats w/ & w/o SW Inj (19 60-sec segs)

Compare $\Omega(f) = 193$ injection, which is visible minute-by-minute.
\[ \Omega(f) = 3.9, 9.6, 19 \text{ injections recovered from full PG} \]

\[ (\Omega(f) = 1.9 \text{ just at threshold of detectability}) \]

Note: injected same random signals w/different amplitudes into same noise
S4 Hardware Injections

- **1024-second** simulated signals injected into LLO & ALLEGRO hardware a total of **nine** times. Simulated all **three** orientations.

- One “round” of three injections had non-const $\Omega_{gw}(f)$

- Other two rounds (“A” & “B”) injected const $\Omega_{gw}(f) = 8100$ → Focus on those

- **Sensitivity** of cross-correlation to injections simulating XARM (“plus”) and YARM (“minus”) is comparable

- “null” injection less correlated b/c of simulated misalignment
Circles: 90% statistical uncertainty (null measurements less sensitive)
90% dashed calib uncertainty “teardrop” around $\Omega_R = 8100$
HW injections recovered consistent w/cal uncertainty
**Extraction of Hardware Injections**

- A−minus
- A−plus
- A−null
- B−minus
- B−plus
- B−null

Circles: 90% statistical uncertainty (null measurements less sensitive)
90% dashed calib uncertainty “teardrop” around $\Omega_R = 8100$

HW injections recovered consistent w/cal uncertainty  

Zoom in on blue box . . .
Circles: 90% statistical uncertainty
90% dashed calib uncertainty “teardrop” around $\Omega_R = 8100$
Systematic offset $<\text{cal uncertainty}$
S4 Preliminary Cross-Correlation Results

Optimally filter looking for \( \Omega_{gw}(f) \equiv \Omega_R \)
Assume \( H_0 = 72 \text{ km/s/Mpc} \) (so \( \Omega_R = h^2_{72} \Omega_{gw}(f) \))

Analyzed non-playground data w/overlapping 60-sec Hann windows:

<table>
<thead>
<tr>
<th>Type</th>
<th>( T_{eff} ) (hrs)</th>
<th>( \Omega_R )</th>
<th>Point Estimate</th>
<th>Error Bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>XARM</td>
<td>181.2</td>
<td>0.61 + 0.25i</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>YARM</td>
<td>114.7</td>
<td>-0.47 + 0.47i</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>non-NULL</td>
<td>295.8</td>
<td>0.31 + 0.31i</td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>NULL</td>
<td>88.2</td>
<td>10.96 – 43.89i</td>
<td>28.62</td>
<td></td>
</tr>
<tr>
<td>all</td>
<td>384.1</td>
<td>0.31 + 0.30i</td>
<td>0.48</td>
<td></td>
</tr>
</tbody>
</table>

No correlation observed

\( \rightarrow \) Convert CC meas of \( 0.31 + 0.30i \) & theor errorbar of 0.48 into upper limit . . .
Constructing Bayesian Posterior PDF

- Formal prior on $\Omega_{gw}(915 \text{ Hz})$
  from Explorer-Nautilus: uniform on $[0, 115]$

- Marginalize likelihood fcn over calibration uncertainty:
  $L_1$ 5% amp, 2° phase; $A_1$ 10% amp, 3° phase.
  (Assume Gaussian prior in $\ln(\text{amp})$ and phase.)
prelim 90% CL UL: $\Omega_R < 1.02 \text{ i.e., } \sqrt{S_{gw}(915 \text{ Hz})} < 1.5 \times 10^{-23} \text{ Hz}^{1/2}$

100× improvement on $\Omega_{gw}(907 \text{ Hz}) < 115 \text{ [} h_{100}^2 \Omega_{gw}(907 \text{ Hz}) < 60 \text{]}$

from NAUTILUS-EXPLORER [Astone et al., A & A 351, 811 (1999)]
LLO-ALLEGRO: Summary

- First stochastic measurement correlating bar w/ifo data; Probes higher frequency band than LLO-LHO: \( \sim 850 - 950 \) Hz

- Diff orientations of ALLEGRO \( \rightarrow \) different stochastic response (Data taken in 3 orientations during S4)

- Preliminary S4 upper limit results from \( \sim 370 \) hrs of data:
  \[ \sqrt{S_{gw}(915 \text{ Hz})} < 1.5 \times 10^{-23} \text{ Hz}^{-1/2} \]
  I.e., \( \Omega_{gw}(915 \text{ Hz}) < 1.02 \left[ h_{100}^2 \Omega_{gw}(915 \text{ Hz}) < 0.53 \right] \),
  \( 100 \times \) better than EXPLORER-NAUTILUS (previous high freq UL)

- Analysis extracts long-time, low-amplitude simulated signals (software injections)

- Hardware inj extracted consistent w/calibration uncertainty
Extra Slides
Overlap Reduction Function

\[ \gamma_{12}(f) = d_{1a}d_{2}^{ab} \frac{5}{4\pi} \iint_{S^2} d^2 \Omega \hat{n} P^{TT}_{ab}(\hat{n}) e^{i2\pi f \hat{n} \cdot \Delta \vec{r}}/c \]

Depends on alignment of detectors (polarization sensitivity)
Frequency dependence from cancellations when \( \lambda \ll \text{distance} \)
→ Widely separated detectors less sensitive at high frequencies

This wave drives LHO & GEO out of phase
Overlap Reduction Function

\[ \gamma_{12}(f) = d_{1ab}d_{2}^{cd} \frac{5}{4\pi} \int S^2 d^2 \Omega \, d_{12}(\hat{n}) P^{TT}_{ab}(\hat{n}) e^{i2\pi f \hat{n} \cdot \Delta \vec{r} / c} \]

Depends on alignment of detectors (polarization sensitivity)
Frequency dependence from cancellations when \( \lambda \lesssim \) distance
→ Widely separated detectors less sensitive at high frequencies

This wave (same \( \lambda \)) drives LHO & GEO in phase
Constructing Posterior PDF

- Overall estimate $\hat{\Omega}_R = x + iy$ has likelihood function (for given actual $\Omega_R = \Omega_{gw}(915 \text{ Hz})$)

$$P(x, y|\Omega_R, \sigma_\Omega) \propto \exp \left( -\frac{|x + iy - \Omega_R|^2}{2\sigma_\Omega^2} \right)$$

- Bayes's theorem gives posterior PDF

$$P(\Omega_R|x, y, \sigma_\Omega) = \frac{P(x, y|\Omega_R, \sigma_\Omega)P(\Omega_R)}{P(x, y|\sigma_\Omega)}$$

$$\propto e^{-\frac{(x-\Omega_R)^2}{2\sigma_\Omega^2}}P(\Omega_R)$$

Note imag part $y$ of pt est factors out
### Marginalization Over Calibration Uncertainty

- Calibration of LLO & ALLEGRO uncertain in amp & phase
- Marginalize over unknown correction factor $e^{\Lambda+i\phi}$:

$$P(x, y|\Omega_R, \sigma_\Omega, \Lambda, \phi) \propto \exp \left( -\frac{|x + iy - \Omega_R e^{\Lambda+i\phi}|^2}{2\sigma_\Omega^2} \right)$$

so the posterior after marginalizing the likelihood function is

$$P(\Omega_R|x, y, \sigma_\Omega) \propto \int_{-\infty}^{\infty} d\Lambda \int_{-\pi}^{\pi} d\phi \exp \left( -\frac{|x + iy - \Omega_R e^{\Lambda+i\phi}|^2}{2\sigma_\Omega^2} \right) P(\Lambda, \phi) P(\Omega_R)$$

which does depend on imag part $y$
Cal marginalization doesn't matter much @ low SNR
Posterior PDF from $\Omega_R=1.929$ injection (no cal marg)
Posterior PDF from $\Omega_R = 19.2901$ injection (no cal marg)
Time-Shift Analyses

• Learned about timing issues via HW injections:
  Time-shift analysis helped resolve issues w/ ALLEGRO timing
  Also revealed sample-and-hold & other digital effects
  in injection system which introduce relative time shift of
  \[ \frac{1}{2 \times 4096 \text{Hz}} - 18 \, \mu s = 104 \, \mu s \]

• Post-processing correction:
  Simulate small timeshift w/ freq-dependent phase shift

\[ Y(f) \rightarrow Y(f) \, e^{i2\pi f \tau} \]

inv FT of CC integrand gives CC values as fcn of time-shift:

\[ Y(\tau) = \int_{f_{\text{min}}}^{f_{\text{max}}} df \, Y(f) \, e^{i2\pi f \tau} \]
Posterior PDF from all HW injections

- dashed line: no cal marg
- solid line: marg over cal
Posterior PDF & 90% conf band from all HW injections