

First Mock LISA Data Challenge: \mathcal{F} -statistic continuous-wave search

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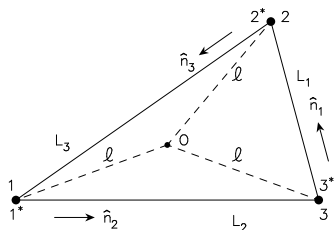
Albert-Einstein-Institut, Potsdam

GWDAAW11 @ Potsdam Dec 19, 2006

Outline

- 1 **Methods and Pipeline**
 - Long-wavelength limit
 - \mathcal{F} -statistic
 - Pipeline
- 2 **Problems encountered**
 - Long-wavelength limit
 - Secondary maxima
- 3 **Outlook and Plans**

TDI 1.0 and Long-wavelength approximation



Long-wavelength limit: $L \ll \frac{\lambda}{2\pi}$
 i.e. $f_{\text{GW}} \ll c/(2\pi L) \sim 10 \text{ mHz}$

👉 expand in $\varepsilon \equiv 2\pi f_{\text{GW}} L/c \ll 1$

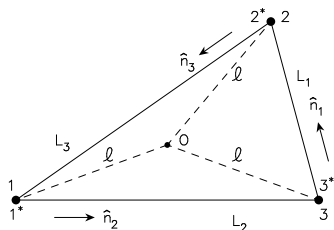
$$X(t) = -4 \frac{L_2 L_3}{c^2} \ddot{h}(t) + \mathcal{O}(\varepsilon^3) \quad [\text{syntheticLISA}]$$

LWL-“strain” $h_X(t) \equiv \frac{1}{2} (\hat{n}_2 \otimes \hat{n}_2 - \hat{n}_3 \otimes \hat{n}_3) : \overleftrightarrow{h}(t)$

“Short Fourier Transforms” (SFTs) $T_{\text{SFT}} = 7 \text{ days}$:

$$\tilde{h}_X(f) = \frac{1}{4(2\pi f L/c)^2} \tilde{X}(f) + \mathcal{O}(\varepsilon) \quad [\text{syntheticLISA}]$$

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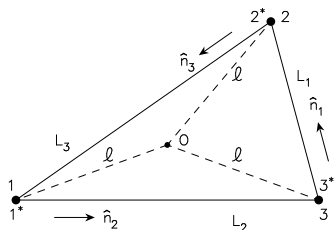
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- *Amplitude parameters:* $\mathcal{A}^{\mu} = \mathcal{A}^{\mu}(h_0, \cos \iota, \psi, \phi_0)$
- *Doppler parameters:* $\lambda = \{\alpha, \delta, f, \dot{f}, \dots\}$

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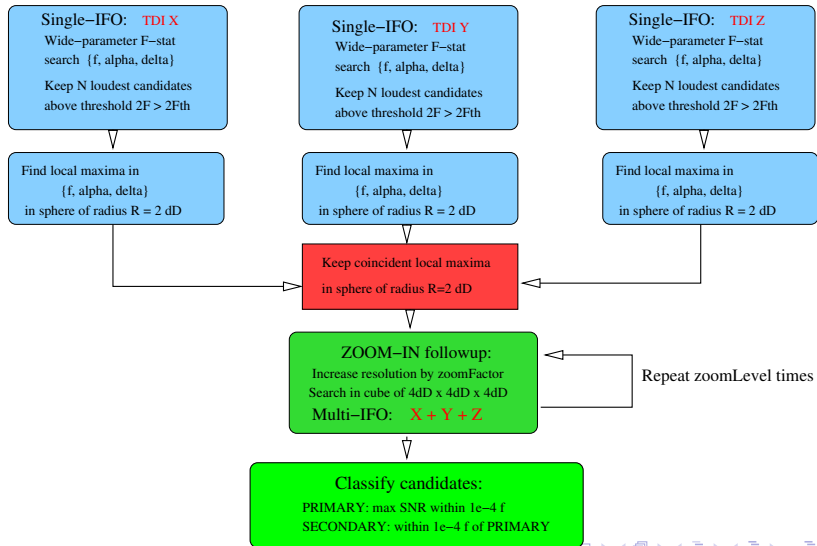
$$x_{\mu}(\lambda) \equiv (x|h_{\mu}), \quad \mathcal{M}_{\mu\nu}(\lambda) \equiv (h_{\mu}|h_{\nu})$$

Jaranowski, Krolak, Schutz 1998: maximize log-likelihood wrt \mathcal{A}^{μ} :

$$\mathcal{A}_{\text{MLE}}^{\mu} = \mathcal{M}^{\mu\nu} x_{\nu} \quad \Rightarrow \text{estimate amplitudes } \mathcal{A}^{\mu} \text{ from data}$$

$$2\mathcal{F}(x; \lambda) = x_{\mu} \mathcal{M}^{\mu\nu} x_{\nu} \quad \Rightarrow \text{need to search only over } \lambda$$

Pipeline

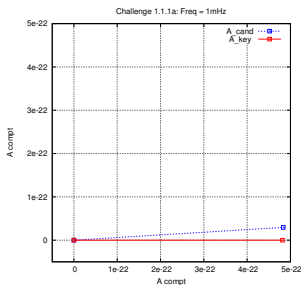


Problems with long-wavelength limit

Challenge 1.1.1a

Challenge 1.1.1b

Challenge 1.1.1c

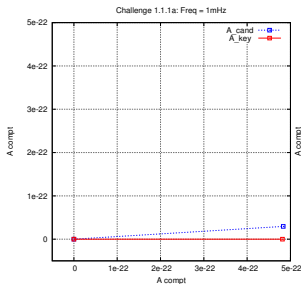


$$f_{GW} \sim 1 \text{ mHz}$$

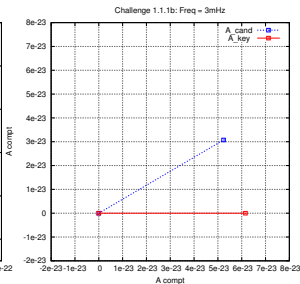
Δf	1.7 nHz
$\Delta \alpha$	0.015
$\Delta \delta$	0.034

Problems with long-wavelength limit

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Challenge 1.1.1b



Challenge 1.1.1c

$$f_{GW} \sim 1 \text{ mHz}$$

$$\Delta f \quad 1.7 \text{ nHz}$$

$$\Delta \alpha \quad 0.015$$

$$\Delta \delta \quad 0.034$$

$$f_{GW} \sim 3 \text{ mHz}$$

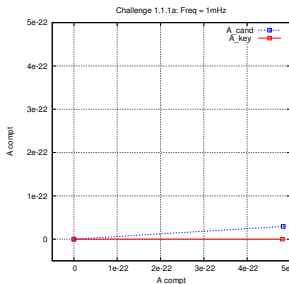
$$\Delta f \quad 0.8 \text{ nHz}$$

$$\Delta \alpha \quad -0.0005$$

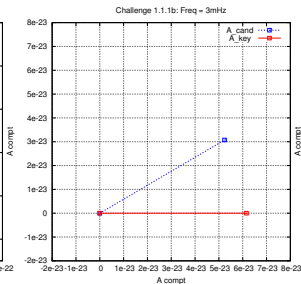
$$\Delta \delta \quad -0.0100$$

Problems with long-wavelength limit

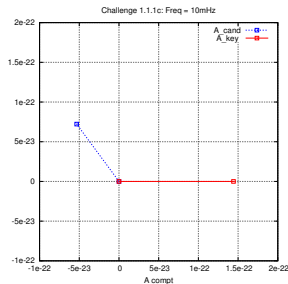
Challenge 1.1.1a



Challenge 1.1.1b



Challenge 1.1.1c



$f_{\text{GW}} \sim 1 \text{ mHz}$

$f_{\text{GW}} \sim 3 \text{ mHz}$

$f_{\text{GW}} \sim 10 \text{ mHz}$

Δf 1.7 nHz

$\Delta \alpha$ 0.015

$\Delta \delta$ 0.034

0.8 nHz

-0.0005

-0.0100

0.2 nHz

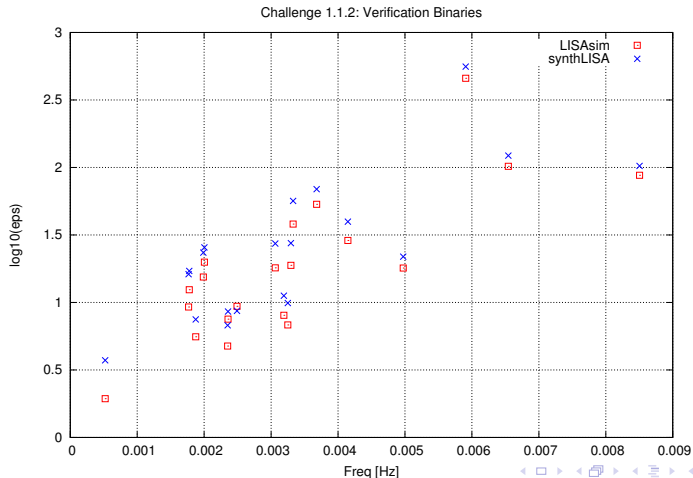
0.0015

0.0051

Problems with long-wavelength limit II

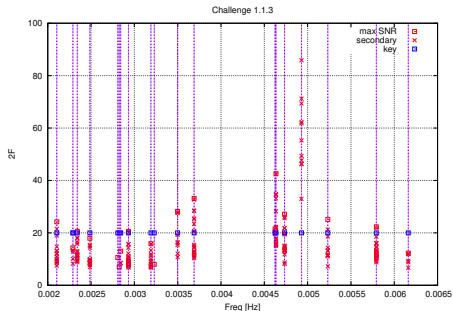
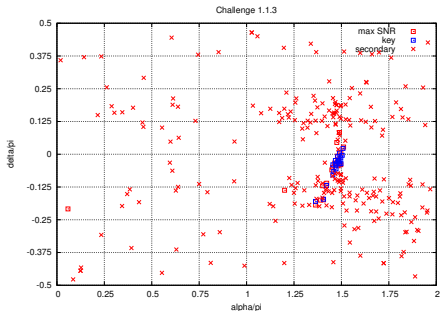
$$\Delta \mathcal{A}^\mu \equiv \mathcal{A}_{\text{key}}^\mu - \mathcal{A}_{\text{MLE}}^\mu$$

“sigma error”: $\epsilon \equiv \sqrt{\mathcal{M}_{\mu\nu} \Delta \mathcal{A}^\mu \Delta \mathcal{A}^\nu}$



Problems with secondary maxima

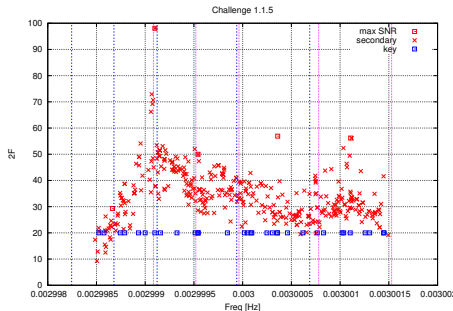
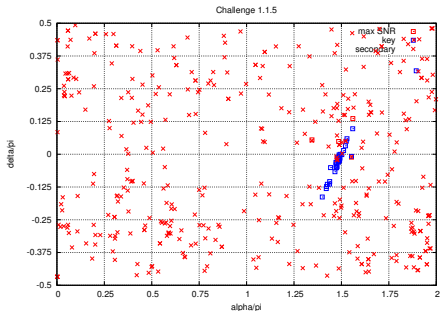
Challenge 1.1.3: Resolvable Binaries:



“clustering” within $\Delta f \sim 10^{-4} f_{\text{GW}}$

Problems with secondary maxima II

Challenge 1.1.5: Strong confusion limit:



👉 need better solution for “clustering”

Outlook and Plans

- adapted `ComputeFStatistic_v2` to LISA/MLDC in LWL:
 - 👉 seems to be working
- Need to go beyond long-wavelength limit for A_{MLE}^{μ}
- Need to fix clustering-problem for secondary maxima:
 - global metric: cluster “close” candidates
 - “subtraction”: *inject* signals with opposite phase
- extend search to SMBH and EMRIs:
 - 👉 search over \dot{f}, \ddot{f} ...

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👉 We'll be back!